UNIT 9

SYMMETRY AND PRACTICAL GEOMETRY

(A) Main Concepts and Results

- A figure is said to have **line symmetry**, if by folding the figure along a line, the left and right parts of it coincide exactly. The line is called the **line** (or **axis) of symmetry** of the figure**.**
- A figure may have no line of symmetry, one line of symmetry, two lines of symmetry, three lines of symmetry and so on.
- Line symmetry is closely related to mirror reflection. The distance of the image of a point (or object) from the line of symmetry (mirror) is the same as that of the point from that line of symmetry.
- Many constructions can be made using different instruments of a geometry box.

(B) Solved Examples

In examples 1 and 2, out of four given options, only one is correct. Write the correct answer.

Example 1: Which of the following letters does not have any line of symmetry?

(A) E (B) T (C) N (D) X

Solution: Correct answer is (C)

Example 2: Which of the following angles cannot be constructed using ruler and compasses?

(A) 75° (B) 15° (C) 135° (D) 85°

Solution: Correct answer is (D)

In examples 3 to 5, fill in the blanks so that the statements are true:

Example 3: If B is the image of A in line *l* and D is the image of C in line *l*, then AC = ________.

Solution: BD

Example 4: In Fig. 9.1, the line segments PQ and RQ have been marked on a line *l* such that PQ = AB and RQ = CD. Then AB – CD = ________.

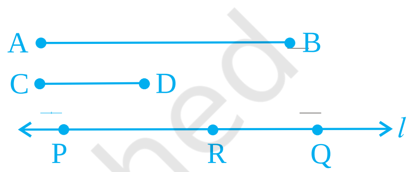


Fig. 9.1

Solution: PR

Example 5: The number of scales in a protractor for measuring the angles is ________.

Solution: Two

In examples 6 and 7, state whether the statements are true or false:

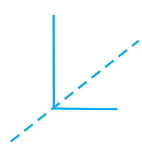
Example 6: Using the set squares 30° – 60° – 90° and 45° – 45° – 90°, we can draw an angle of 75°.

Solution: True. (Since 75° = 45° + 30°)

Example 7: A circle has only 8 lines of symmetry.

Solution: False (A circle has infinitely many lines of symmetry).

Example 8. Write the letters of the word ALGEBRA which have no line of symmetry.

Solution: The letters L, G and R have no line of symmetry. (Do you see why the dotted line is not the line of symmetry in  ?)

Example 9: Draw a line segment equal to the sum of two line segments given in Fig. 9.2

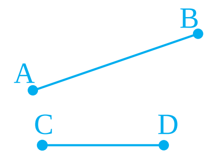


Fig. 9.2

Solution: 1. Draw a line l and on it, cut a line segment PQ = AB, using compasses. (Fig. 9.3)

Fig. 9.3

2. With Q as centre and CD as radius, draw an arc to cut a line segment QS = CD on l as shown in Fig. 9.4. Then, line segment PS is equal to the sum of AB and CD, i.e., PS = AB + CD

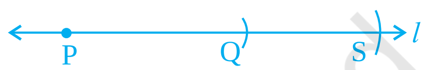


Fig. 9.4

Example 10. Draw an angle equal to the difference of two angles given in Fig. 9.5.

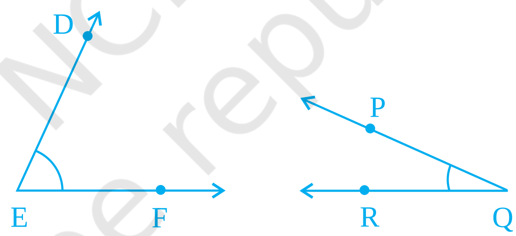


Fig. 9.5

Solution: 1. Draw an angle ABC equal to ∠DEF (as ∠DEF > ∠PQR), using ruler and compasses.

2. With BC as one of the arms, draw an angle SBC equal to ∠PQR such that BS is in the interior of ∠ABC as shown in Fig. 9.6. Then, ∠ABS is the required angle which is equal to ∠DEF – ∠PQR.

[Note: For making ∠ABS = ∠DEF – ∠PQR, how will you draw ray BS?]

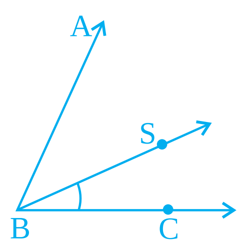


Fig. 9.6

Example 11. Complete Fig. 9.7 so that *l* is the line of symmetry of the completed figure.

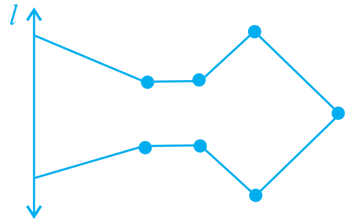


Fig. 9.7

Solution: The figure can be completed as shown in Fig. 9.8, by drawing the points symmetric to different corners(points) with respect to line *l*.

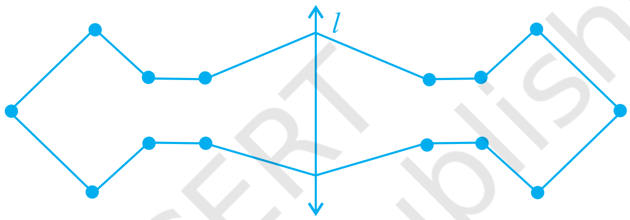
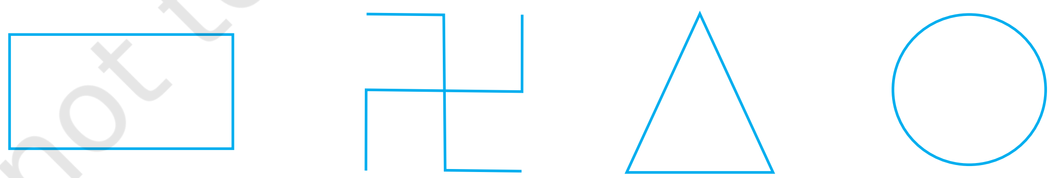


Fig. 9.8

(C) Exercise

In questions 1 to 17, out of the given four options, only one is correct. Write the correct answer.

1. In the following figures, the figure that is not symmetric with respect to any line is:



(i) (ii) (iii) (iv)

(A) (i) (B) (ii) (C) (iii) (D) (iv)

2. The number of lines of symmetry in a scalene triangle is

(A) 0 (B) 1 (C) 2 (D) 3

3. The number of lines of symmetry in a circle is

(A) 0 (B) 2 (C) 4 (D) more than 4

4. Which of the following letters does not have the vertical line of symmetry?

(A) M (B) H (C) E (D) V

5. Which of the following letters have both horizontal and vertical lines of symmetry?

(A) X (B) E (C) M (D) K

6. Which of the following letters does not have any line of symmetry?

(A) M (B) S (C) K (D) H

7. Which of the following letters has only one line of symmetry?

(A) H (B) X (C) Z (D) T

8. The instrument to measure an angle is a

(A) Ruler (B) Protractor (C) Divider (D) Compasses

9. The instrument to draw a circle is

(A) Ruler (B) Protractor (C) Divider (D) Compasses

10. Number of set squares in the geometry box is

(A) 0 (B) 1 (C) 2 (D) 3

11. The number of lines of symmetry in a ruler is

(A) 0 (B) 1 (C) 2 (D) 4

12. The number of lines of symmetry in a divider is

(A) 0 (B) 1 (C) 2 (D) 3

13. The number of lines of symmetry in compasses is

(A) 0 (B) 1 (C) 2 (D) 3

14. The number of lines of symmetry in a protractor is

(A) 0 (B) 1 (C) 2 (D) more than 2

15. The number of lines of symmetry in a 45° - 45° - 90° set-square is

(A) 0 (B) 1 (C) 2 (D) 3

16. The number of lines of symmetry in a 30° - 60° - 90° set square is

(A) 0 (B) 1 (C) 2 (D) 3

17. The instrument in the geometry box having the shape of a triangle is called a

(A) Protractor

(B) Compasses

(C) Divider

(D) Set-square

In questions 18 to 42, fill in the blanks to make the statements true.

18. The distance of the image of a point (or an object) from the line of symmetry (mirror) is ________ as that of the point (object) from the line (mirror).

19. The number of lines of symmetry in a picture of Taj Mahal is _______.

20. The number of lines of symmetry in a rectangle and a rhombus are ______ (equal/unequal).

21. The number of lines of symmetry in a rectangle and a square are______ (equal/unequal).

22. If a line segment of length 5cm is reflected in a line of symmetry (mirror), then its reflection (image) is a ______ of length _______.

23. If an angle of measure 80° is reflected in a line of symmetry, then the reflection is an ______ of measure _______.

24. The image of a point lying on a line l with respect to the line of symmetry l lies on _______.

25. In Fig. 9.10, if B is the image of the point A with respect to the line l and P is any point lying on l, then the lengths of line segments PA and PB are _______.

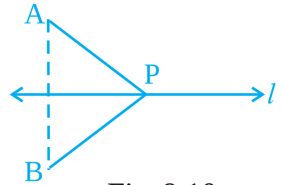


Fig. 9.10

26. The number of lines of symmetry in Fig. 9.11 is_________.

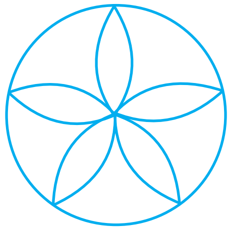


Fig. 9.11

27. The common properties in the two set-squares of a geometry box are that they have a _________ angle and they are of the shape of a _________.

28. The digits having only two lines of symmetry are________ and _________.

29. The digit having only one line of symmetry is _________.

30. The number of digits having no line of symmetry is________.

31. The number of capital letters of the English alphabets having only vertical line of symmetry is_______.

32. The number of capital letters of the English alphabets having only horizontal line of symmetry is_______.

33. The number of capital letters of the English alphabets having both horizontal and vertical lines of symmetry is_______.

34. The number of capital letters of the English alphabets having no line of symmetry is_________.

35. The line of symmetry of a line segment is the _______ bisector of the line segment.

36. The number of lines of symmetry in a regular hexagon is _________.

37. The number of lines of symmetry in a regular polygon of n sides is_______.

38. A protractor has _________ line/lines of symmetry.

39. A 30° - 60° - 90° set-square has ________ line/lines of symmetry.

40. A 45° - 45° - 90° set-square has _______ line/lines of symmetry.

41. A rhombus is symmetrical about _________.

42. A rectangle is symmetrical about the lines joining the _________ of the opposite sides.

In questions 43 - 61, state whether the statements are true (T) or false (F).

43. A right triangle can have at most one line of symmetry.

44. A kite has two lines of symmetry.

45. A parallelogram has no line of symmetry.

46. If an isosceles triangle has more than one line of symmetry, then it need not be an equilateral triangle.

47. If a rectangle has more than two lines of symmetry, then it must be a square.

48. With ruler and compasses, we can bisect any given line segment.

49. Only one perpendicular bisector can be drawn to a given line segment.

50. Two perpendiculars can be drawn to a given line from a point not lying on it.

51. With a given centre and a given radius, only one circle can be drawn.

52. Using only the two set-squares of the geometry box, an angle of 40° can be drawn.

53. Using only the two set-squares of the geometry box, an angle of 15° can be drawn.

54. If an isosceles triangle has more than one line of symmetry, then it must be an equilateral triangle.

55. A square and a rectangle have the same number of lines of symmetry.

56. A circle has only 16 lines of symmetry.

57. A 45° - 45° - 90° set-square and a protractor have the same number of lines of symmetry.

58. It is possible to draw two bisectors of a given angle.

59. A regular octagon has 10 lines of symmetry.

60. Infinitely many perpendiculars can be drawn to a given ray.

61. Infinitely many perpendicular bisectors can be drawn to a given ray.

62. Is there any line of symmetry in the Fig. 9.12? If yes, draw all the lines of symmetry.

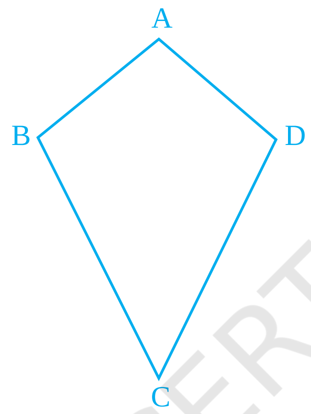


Fig. 9.12

63. In Fig. 9.13, PQRS is a rectangle. State the lines of symmetry of the rectangle.

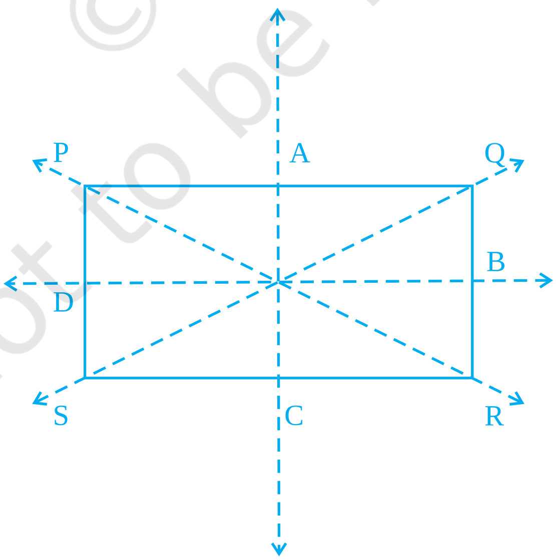


Fig. 9.13

64. Write all the capital letters of the English alphabets which have more than one lines of symmetry.

65. Write the letters of the word 'MATHEMATICS' which have no line of symmetry.

66. Write the number of lines of symmetry in each letter of the word 'SYMMETRY'.

67. Match the following:

Shape	Number of lines of symmetry
(i) Isosceles triangle	(a) 6
(ii) Square	(b) 5
(iii) Kite	(c) 4
(iv) Equilateral triangle	(d) 3
(v) Rectangle	(e) 2
(vi) Regular hexagon	(f) 1
(vii) Scalene triangle	(g) 0

68. Open your geometry box. There are some drawing tools. Observe them and complete the following table:

Name of the tool	Number of lines of symmetry
(i) The Ruler	________
(ii) The Divider	________
(iii) The Compasses	________
(iv) The Protactor	________
(v) Triangular piece with two equal sides	________
(vi) Triangular piece with unequal sides	________

69. Draw the images of points A and B in line l of Fig. 9.14 and name them as A′ and B′ respectively. Measure AB and A′ B′. Are they equal?

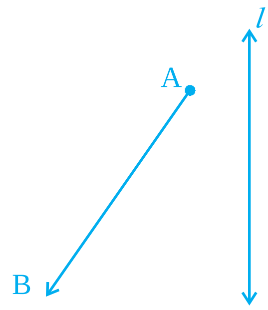


Fig. 9.14

70. In Fig. 9.15, the point C is the image of point A in line l and line segment BC intersects the line l at P.

(a) Is the image of P in line l the point P itself?

(b) Is PA = PC?

(c) Is PA + PB = PC + PB?

(d) Is P that point on line l from which the sum of the distances of points A and B is minimum?

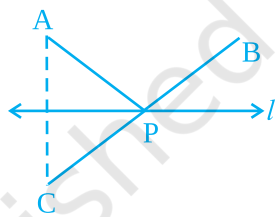


Fig. 9.15

71. Complete the figure so that line l becomes the line of symmetry of the whole figure (Fig. 9.16).

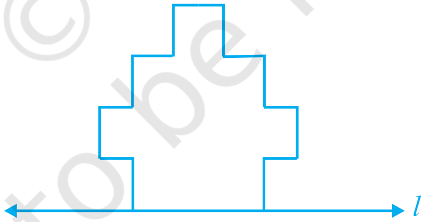


Fig. 9.16

72. Draw the images of the points A, B and C in the line m (Fig. 9.17). Name them as A′, B′ and C′, respectively and join them in pairs. Measure AB, BC, CA, A′B′, B′C′ and C′A′. Is AB = A′B′, BC = B′C′ and CA = C′A′?

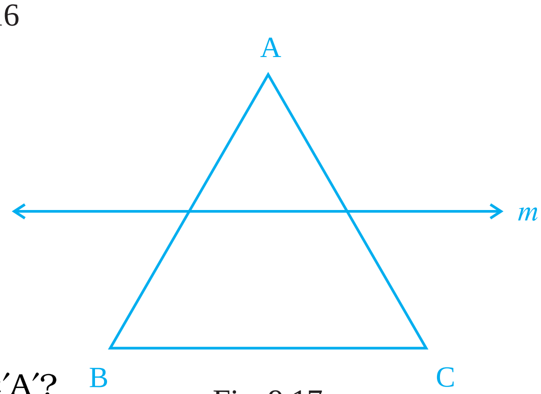


Fig. 9.17

73. Draw the images P′, Q′ and R′ of the points P, Q and R, respectively in the line *n* (Fig. 9.18). Join P′ Q′ and Q′ R′ to form an angle P′ Q′ R′. Measure ∠PQR and ∠P′Q′R′. Are the two angles equal?

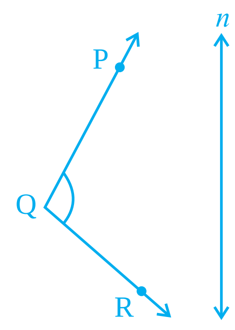


Fig. 9.18

74. Complete Fig. 9.19 by taking *l* as the line of symmetry of the whole figure.

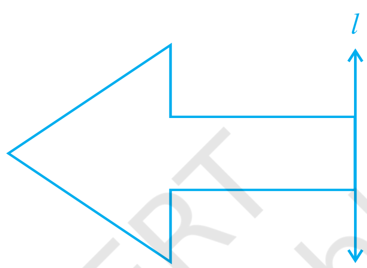


Fig. 9.19

75. Draw a line segment of length 7cm. Draw its perpendicular bisector, using ruler and compasses.

76. Draw a line segment of length 6.5cm and divide it into four equal parts, using ruler and compasses.

77. Draw an angle of 140° with the help of a protractor and bisect it using ruler and compasses.

78. Draw an angle of 65° and draw an angle equal to this angle, using ruler and compasses.

79. Draw an angle of 80° using a protractor and divide it into four equal parts, using ruler and compasses.Check your construction by measurement.

80. Copy Fig. 9.20 on your notebook and draw a perpendicular to *l* through P, using (i) set squares (ii) Protractor (iii) ruler and compasses. How many such perpendiculars are you able to draw?

Fig. 9.20

81. Copy Fig. 9.21 on your notebook and draw a perpendicular from P to line *m*, using (i) set squares (ii) Protractor (iii) ruler and compasses. How many such perpendiculars are you able to draw?

Fig. 9.21

82. Draw a circle of radius 6cm using ruler and compasses. Draw one of its diameters. Draw the perpendicular bisector of this diameter. Does this perpendicular bisector contain another diameter of the circle?

83. Bisect $\angle XYZ$ of Fig. 9.22

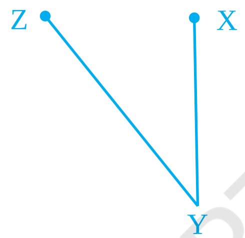


Fig. 9.22

84. Draw an angle of 60° using ruler and compasses and divide it into four equal parts. Measure each part.

85. Bisect a straight angle, using ruler and compasses. Measure each part.

86. Bisect a right angle, using ruler and compasses. Measure each part. Bisect each of these parts. What will be the measure of each of these parts?

87. Draw an angle ABC of measure 45°, using ruler and compasses. Now draw an angle DBA of measure 30°, using ruler and compasses as shown in Fig. 9.23. What is the measure of $\angle DBC$?

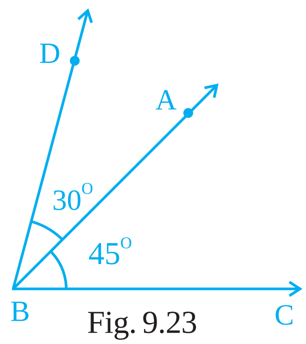


Fig. 9.23

88. Draw a line segment of length 6cm. Construct its perpendicular bisector. Measure the two parts of the line segment.

89. Draw a line segment of length 10cm. Divide it into four equal parts. Measure each of these parts.

(D) Activities

Activity 1: Make three different ink blot devils in your notebook and mark their line of symmetry.

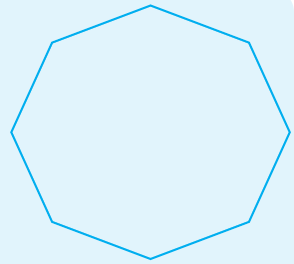


Fig. 9.24

Activity 2: Draw all the lines of symmetry of Fig. 9.24 by paper folding.

Activity 3: Draw an angle of 15° by first drawing an angle of 60° and then an angle of 45°, using ruler and compasses.

Activity 4: Using ruler and compasses draw an angle of 90° and in its interior, draw two rays with the initial point of each as the vertex of the angle so that each of the three angles so formed is of 30° (See Fig. 9.25).

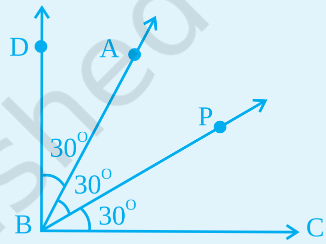


Fig. 9.25

Activity 5: Draw an angle of 45° and in its interior, draw two rays to form three angles each of measure 15°, using ruler and compasses.

Activity 6: Draw an angle of 135° and in its interior, draw two rays to form three angles each of equal measure, using ruler and compasses.

Activity 7: Draw the perpendicular bisectors of BC, CA and AB (Fig. 9.26). What do you observe?

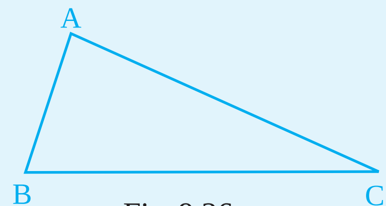


Fig. 9.26

Activity 8: Bisect AE and CE by drawing up their perpendicular bisectors in (Fig. 9.27).

Let P be the point of intersection of these perpendicular bisectors check whether

PA = PE, PE = PC

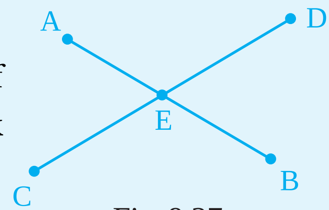


Fig. 9.27

Activity 9: Bisect BC and AB by drawing their perpendicular bisectors (Fig. 9.28). Make the point of intersecton as P. Check whether PA = PB = PC

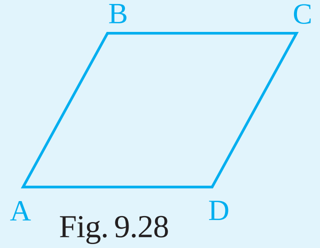


Fig. 9.28

Activity 10: Draw two line segments of lengths 8cm and 6cm. Using these line segments, construct a line segment of length (8 + 6)cm.

Activity 11: Draw two line segments of lengths 3cm and 5cm. Construct line segments of the following lengths using these line segments:

(a) 6cm (b) 15cm (c) (3+5)cm

(d) (6+5)cm (e) (9 – 5)cm (f) (5 – 3)cm

Activity 12: Draw two line segments of lengths 3cm and 6cm. Construct line segments, equal to the following lengths, using these line segments.

(a) $\frac{3+6}{2}$ cm (b) $\frac{6}{2}$ cm (c) $\frac{2(3)+6}{2}$ cm

Activity 13: Drop perpendiculars from D to AB and from D to AC (Fig. 9.29).

Fig. 9.29

Activity 14: O is the centre of the circle (Fig. 9.30). Drop perpendicular from B on CA. Where does it meet CA?

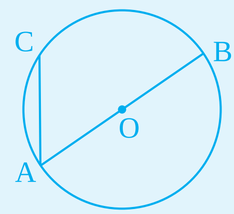


Fig. 9.30

Activity 15: Copy the figure and bisect ∠A and ∠B (Fig. 9.31). Let the bisectors meet at some point P. Measure angle ∠APB.

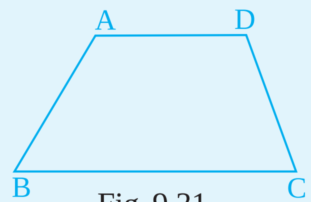


Fig. 9.31

Activity 16:

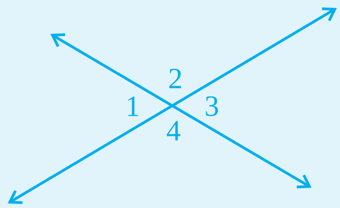


Fig. 9.32

(a) Bisect angle 1 and angle 2 (Fig. 9.32).

(b) Measure the angle between these bisectors.

(c) Now bisect angle 3 and angle 4.

(d) Measure the angle formed between these bisectors.

(e) What do you obeserve from (b) and (d)? Can you conclude something?

Activity 17: Construct an angle equal to $1\frac{1}{2}$ times the $\angle PQR$ of Fig. 9.33, using ruler and compasses.

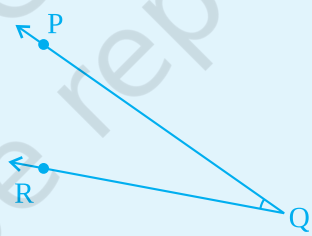


Fig. 9.33

Activity 18: Bisect angle A, angle B and angle C (Fig. 9.34). What do you observe?

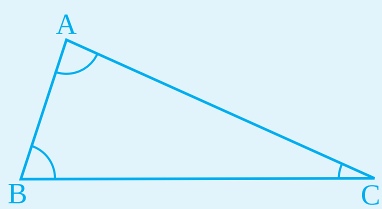


Fig. 9.34