

# ANSWERS

## 1.3 EXERCISE

1.  $(b,b), (c,c), (a,c)$
2.  $[-5,5]$
3.  $4x^2 + 4x - 1$
4.  $f^{-1}(x) = \frac{x+3}{2}$
5.  $f^{-1}\{(b,a), (d,b), (a,c), (c,d)\}$
6.  $f(f(x)) = x^4 - 6x^3 + 10x^2 - 3x$
7.  $\alpha=2, \beta=-1$
8. (i) represents function which is surjective but not injective  
(ii) does not represent function.
9.  $fog = \{(2,5), (5,2), (1,5)\}$
12. (i)  $f$  is not function (ii)  $g$  is function (iii)  $h$  is function (iv)  $k$  is not function
14.  $\left[\frac{1}{3}, 1\right]$
17. Domain of  $R = \{1, 2, 3, 4, \dots, 20\}$  and  
Range of  $R = \{1, 3, 5, 7, 9, \dots, 39\}$ .  $R$  is neither reflective, nor symmetric and nor transitive.
21. (i)  $f$  is one-one but not onto, (ii)  $g$  is neither one-one nor onto (iii)  $h$  is bijective,  
(iv)  $k$  is neither one-one nor onto.
22. (i) transitive (ii) symmetric (iii) reflexive, symmetric and transitive (iv) transitive.
23.  $[(2,5)] = \{(1,4), (2,5), (3,6), (4,7), (5,8), (6,9)\}$

**25.** (i)  $(fog)(x) = 4x^2 - 6x + 1$

(ii)  $(gof)(x) = 2x^2 + 6x - 1$

(iii)  $(fof)(x) = x^4 + 6x^3 + 14x^2 + 15x + 5$

(iv)  $(gog)(x) = 4x - 9$

**26.** (ii) & (iv)

**27.** (i)

**28.** C

**29.** B

**30.** D

**31.** B

**32.** B

**33.** A

**34.** C

**35.** C

**36.** B

**37.** D

**38.** A

**39.** B

**40.** B

**41.** A

**42.** A

**43.** C

**44.** B

**45.** D

**46.** A

**47.** B

**48.**  $R = \{(3,8), (6,6), (9,4), (12,2)\}$

**49.**  $R = \{(1,1), (1,2), (2,1), (2,2), (2,3), (3,2), (3,3), (3,4), (4,3), (4,4), (5,5)\}$

**50.**  $gof = \{(1,3), (3,1), (4,3)\}$  and  $fog = \{(2,5), (5,2), (1,5)\}$

**51.**  $(fof)(x) = \frac{x}{\sqrt{3x^2 + 1}}$

**52.**  $f^{-1}(x) = 7 + (4 - x)^{\frac{1}{3}}$

**53.** False

**54.** False

**55.** False

**56.** False

**57.** True

**58.** False

**59.** False

**60.** True

**61.** False

**62.** False

### 2.3 EXERCISE

**1.** 0

**2.** -1

**4.**  $\frac{-}{12}$

**5.**  $-\frac{1}{3}$

**7.** 0, -1

**8.**  $\frac{14}{15}$

**11.**  $\frac{-3}{4}, \frac{3}{4}$

**13.**  $\tan^{-1} \frac{4}{3} - x$

**17.**  $\frac{\pi}{4}$

**19.**  $\frac{a_n - a_1}{1 + a_1 a_n}$

**20.** C

**21.** D

**22.** B

**23.** D

**24.** A

**25.** A

**26.** B

**27.** C

**28.** A

**29.** B

**30.** A

**31.** D

**32.** D

**33.** B

**34.** A

**35.** C

**36.** A

**37.** A

**38.**  $\frac{2}{3}$

**39.**  $\frac{2}{5}$

**40.**  $\sqrt{3}$

**41.**  $\phi$

**42.**  $\bar{\frac{1}{3}}$

**43.**  $\frac{2}{3}$

**44.** 0

**45.** 1

**46.**  $-2, 2$

**47.**  $xy > -1$

**48.**  $-\cot^{-1} x$

**49.** False

**50.** False

**51.** True

**52.** True

**53.** True

**54.** False

**55.** True

### 3.3 EXERCISE

**1.**  $28 \times 1, 1 \times 28, 4 \times 7, 7 \times 4, 14 \times 2, 2 \times 14$ . If matrix has 13 elements then its order will be either  $13 \times 1$  or  $1 \times 13$ .

**2.** (i)  $3 \times 3$ , (ii) 9, (iii)  $a_{23} = x^2 - y, a_{31} = 0, a_{12} = 1$

**3.** (i)  $\begin{bmatrix} 1 & 9 \\ 2 & 2 \\ 0 & 2 \end{bmatrix}$       (ii)  $\begin{bmatrix} 1 & 4 \\ -1 & 2 \end{bmatrix}$

**4.**  $\begin{bmatrix} e^x \sin x & e^x \sin 2x \\ e^{2x} \sin x & e^{2x} \sin 2x \\ e^{3x} \sin x & e^{3x} \sin 2x \end{bmatrix}$

**5.**  $a = 2, b = 2$

**6.** Not possible

**7.** (i)  $X + Y = \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$       (ii)  $2X - 3Y = \begin{bmatrix} 0 & -1 & 1 \\ -11 & -10 & -18 \end{bmatrix}$

(iii)  $Z = \begin{bmatrix} -5 & -2 & 2 \\ -12 & 0 & -1 \end{bmatrix}$

8.  $x = 4$

10.  $-2, -14$

11.  $A^{-1} = \frac{-1}{7} \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$

12.  $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

13.  $A = [-1 \ 2 \ 1]$

15.  $AB = \begin{bmatrix} 12 & 9 \\ 12 & 15 \end{bmatrix} BA = \begin{bmatrix} 9 & 6 & 12 \\ 7 & 8 & 16 \\ 4 & 5 & 10 \end{bmatrix}$

18.  $x = 1, y = 2$

19.  $X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}, Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$

20.  $\begin{bmatrix} k \\ 2k \end{bmatrix}, \begin{bmatrix} k & k \\ 2k & 2k \end{bmatrix}$  etc.

where  $k$  is a real number

24.  $A = [-4]$

30. True when  $AB = BA$

37. (i)  $\frac{1}{22} \begin{bmatrix} 7 & -3 \\ 5 & 1 \end{bmatrix}$  (ii) not possible

38.  $x = 2, y = 4$  or  $x = 4, y = 2, z = -6, w = 4$

39.  $\begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$

40.  $A^3 = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$

41.  $a = 2, b = 4, c = 1, d = 3$

42.  $\begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$

43.  $\begin{bmatrix} 18 & 8 \\ 16 & 18 \end{bmatrix}$

44. True for all real values of  $\alpha$

45.  $a = -2, b = 0, c = -3$

**50.**  $x = \pm \frac{1}{\sqrt{2}}, y = \pm \frac{1}{\sqrt{6}}, z = \pm \frac{1}{\sqrt{3}}$

**51.** (i)  $\begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$  (ii) inverse does not exist (iii)  $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

**52.** 
$$\begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

- |  |                      |                                  |                        |
|--|----------------------|----------------------------------|------------------------|
| <b>53.</b> A   | <b>54.</b> D         | <b>55.</b> B                     | <b>56.</b> D           |
| <b>57.</b> D   | <b>58.</b> D         | <b>59.</b> A                     | <b>60.</b> B           |
| <b>61.</b> C   | <b>62.</b> D         | <b>63.</b> A                     | <b>64.</b> A           |
| <b>65.</b> D   | <b>66.</b> D         | <b>67.</b> A                     | <b>68.</b> Null matrix |
| <b>69.</b> Skew symmetric matrix   |                      | <b>70.</b> -1                    | <b>71.</b> 0           |
| <b>72.</b> Rectangular matrix  |                      | <b>73.</b> Distributive          |                        |
| <b>74.</b> Symmetrix matrix  |                      | <b>75.</b> Symmetrix matrix      |                        |
| <b>76.</b> (i) $B'A'$ (ii) $kA$ (iii) $k(A-B)$   |                      | <b>77.</b> Skew Symmetric matrix |                        |
| <b>78.</b> (i) Skew symmetric matrix<br>(ii) neither symmetric nor skew symmetric matrix |                      |                                  |                        |
| <b>79.</b> Symmetric matrix  | <b>80.</b> $AB = BA$ | <b>81.</b> does not exist        |                        |
| <b>82.</b> False   | <b>83.</b> False     | <b>84.</b> False                 | <b>85.</b> True        |
| <b>86.</b> True  | <b>87.</b> False     | <b>88.</b> False                 | <b>89.</b> True        |
| <b>90.</b> False   | <b>91.</b> False     | <b>92.</b> False                 | <b>93.</b> False       |
| <b>94.</b> True  | <b>95.</b> False     | <b>96.</b> True                  | <b>97.</b> False       |
| <b>98.</b> True  | <b>99.</b> False     | <b>100.</b> True                 | <b>101.</b> True       |

**4.3 EXERCISE**

- 1.**  $x^3 - x^2 + 2$       **2.**  $a^2(a + x + y + z)$       **3.**  $2x^3y^3z^3$   
**4.**  $3(x + y + z)(xy + yz + zx)$       **5.**  $16(3x + 4)$       **6.**  $(a + b + c)^3$
- 12.**  $= n$  or  $n = (-1)^n \left(\frac{\pi}{6}\right)$       **13.**  $x = 0, -12$       **18.**  $x = 0, y = -5, z = -3$
- 19.**  $x = 1, y = 1, z = 1$       **20.**  $x = 2, y = -1, z = 4$
- 24.** C      **25.** C      **26.** B      **27.** D  
**28.** C      **29.** A      **30.** A      **31.** A  
**32.** C      **33.** D      **34.** D      **35.** D
- 36.** B      **37.** C      **38.**  $27|A|$       **39.**  $\frac{1}{|A|}$
- 40.** Zero      **41.**  $\frac{1}{2}$       **42.**  $(A^{-1})^2$       **43.** 9  
**44.** Value of the determinant      **45.**  $x = 2, y = 7$   
**46.**  $(y - z)(z - x)(y - x + xyz)$       **47.** Zero      **48.** True  
**49.** False      **50.** False      **51.** True      **52.** True  
**53.** True      **54.** False      **55.** True      **56.** True  
**57.** True      **58.** True

**5.3 EXERCISE**

- 1.** Continuous at  $x = 1$       **2.** Discontinuous      **3.** Discontinuous      **4.** Continuous  
**5.** Discontinuous      **6.** Continuous      **7.** Continuous      **8.** Discontinuous
- 9.** Continuous      **10.** Continuous      **11.**  $k = \frac{7}{2}$       **12.**  $k = \frac{1}{2}$
- 13.**  $k = -1$       **14.**  $k = \pm 1$       **16.**  $a = 1, b = -1$
- 17.** Discontinuous at  $x = -2$  and  $x = \frac{-5}{2}$       **18.** Discontinuous at  $x = 1, \frac{1}{2}$  and 2  
**20.** Not differentiable at  $x = 2$       **21.** Differentiable at  $x = 0$   
**22.** Not differentiable at  $x = 2$       **25.**  $-(\log 2) \cdot \sin 2x \cdot 2^{\cos^2 x}$

**26.**  $\frac{8^x}{x^8} \left[ \log 8 - \frac{8}{x} \right]$  **27.**  $\frac{1}{\sqrt{x^2 + a}}$  **28.**  $\frac{5}{x \log(x^5) \log(\log x^5)}$

**29.**  $\frac{\cos \sqrt{x}}{2\sqrt{x}} - \frac{\sin 2\sqrt{x}}{2\sqrt{x}}$  **30.**  $n(2ax+b)\sin^{n-1}(ax^2+bx+c)\cos(ax^2+bx+c)$

**31.**  $\frac{-1}{2\sqrt{x+1}} \sin(\tan \sqrt{x+1}) \sec^2(\sqrt{x+1})$

**32.**  $2x \cos(x)^2 + 2x \sin(2x^2) + \sin 2x$  **33.**  $\frac{-1}{2\sqrt{x}(x+1)}$

**34.**  $(\sin x)^{\cos x} \left[ \frac{\cos^2 x}{\sin x} - \sin x \cdot \log \sin x \right]$  **35.**  $\sin^{mx} x \cos^n x (-n \tan x + m \cot x)$

**36.**  $(x+1)(x+2)^2(x+3)^3[9x^2+34x+29]$

**37.**  $-1$  **38.**  $\frac{1}{2}$  **39.**  $\frac{1}{2}$  **40.**  $-1$

**41.**  $\frac{-3}{\sqrt{1-x^2}}$  **42.**  $\frac{3a}{a^2+x^2}$  **43.**  $\frac{-x}{\sqrt{1-x^4}}$  **44.**  $\frac{t^2+1}{t^2-1}$

**45.**  $e^{-2} \left( \frac{-3+2+1}{-3+2+-1} \right)$  **46.**  $\cot \theta$  **47.**  $1$

**48.**  $t$  **51.**  $-\frac{1}{\sqrt{3}}$  **52.**  $\frac{\tan x - x}{\sin^2 x}$  **53.**  $\frac{1}{2}$

**54.**  $\frac{2xy^2 - y^3 \cos(xy) - y}{xy^2 \cos(xy) - x + y^2}$  **55.**  $\frac{y - \sec(x+y) \tan(x+y)}{\sec(x+y) \tan(x+y) - x}$

**56.**  $\frac{-x}{y}$  **57.**  $\frac{y - 4x^3 - 4xy^2}{4yx^2 + 4y^3 - x}$  **64.**  $-2 \sin y \cos^3 y$

**70.** Not applicable since  $f$  is not differentiable at  $x = 1$

**71.**  $(\pi, -2)$

**72.**  $(2, -4)$

**77.**  $\left(\frac{7}{2}, \frac{1}{4}\right)$

**78.**  $\left(\frac{3}{2}, 0\right)$

**79.**  $p=3, q=5$

**82.**  $x^{\tan x} \left( \sec^2 x \log x + \frac{\tan x}{x} \right) + \frac{x}{\sqrt{2} \sqrt{x^2+1}}$  **83.** D

**84.** C

**85.** B

**86.** A

**87.** A

**88.** A

**89.** C

**90.** B

**91.** B

**92.** A

**93.** A

**94.** B

**95.** A

**96.** B

**97.**  $|x| + |x-1|$

**98.**  $\frac{2}{3x}$

**99.**  $\frac{-1}{\sqrt{2}}$

**100.**  $\left(\frac{\sqrt{3}+1}{2}\right)$

**101.** -1

**102.** False

**103.** True

**104.** True

**105.** True

**106.** False

### 6.3 EXERCISE

**3.** 8 m/s

**4.**  $\left(\sqrt{2-\sqrt{2}}\right)v$  unit/sec. **5.**  $= \frac{-1}{3}$

**6.** 31.92

**7.**  $0.018\pi \text{cm}^3$

**8.**  $2\frac{2}{3}$  m/s towards light, -1 m/s

**9.** 2000 litres/s, 3000 litre/s

**11.**  $2x^3 - 3x + 1$

**12.**  $k^2 = 8$

**14.** (4, 4)

**15.**  $\tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$  **17.**  $x + 3y = \pm 8$

**18.** (3, 2), (-1, 2) **23.** (1, -16), max. slope = 12

**26.**  $x = 1$  is the point of local maxima; local maximum = 0

$x = 3$  is the point of local minima; local minimum = -28

$x = 0$  is the point of inflection.

**27.** Rs 100

**30.** 6cm, 12 cm,  $864\pi \text{cm}^3$

31. 1:1

33. Rs 1920

$$34. \frac{2}{3}x^3 \left(1 + \frac{2}{27}\right)$$

35. C

36. B

37. A

38. C

39. D

40. A

41. A

42. D

43. B

44. B

45. C

46. B

47. D

48. A

49. B

50. C

51. A

52. C

53. B

54. C

55. B

56. A

57. B

58. B

59. C

60. (3, 34)

$$61. x + y = 0$$

$$62. (-\infty, -1)$$

63.  $(1, \infty)$ 

$$64. 2\sqrt{ab}$$

### 7.3 EXERCISE

$$3. \frac{x^2}{2} - x + 3 \log|x+1| + c$$

$$4. \frac{x^3}{3} + c \quad 5. \log|x+\sin x| + c$$

$$6. \tan \frac{x}{2} + C$$

$$7. \frac{\tan^5 x}{5} + \frac{\tan^3 x}{3} + c \quad 8. x + c$$

$$9. -2 \cos \frac{x}{2} + 2 \sin \frac{x}{2} + c$$

$$10. 2 \left[ \frac{x\sqrt{x}}{3} - \frac{x}{2} + \sqrt{x} - \log|\sqrt{x}+1| \right] + c$$

$$11. -a \left[ \cos^{-1} \left( \frac{x}{a} \right) + \sqrt{1 - \frac{x^2}{a^2}} \right] + c$$

$$12. \frac{4}{3} \left[ x^{3/4} - \log \left| 1 + x^{\frac{3}{4}} \right| \right] + c$$

$$13. \frac{-1}{3} \left( 1 + \frac{1}{x^2} \right)^{\frac{3}{2}} + c$$

$$14. \frac{1}{3} \sin^{-1} \frac{3x}{4} + c$$

$$15. \frac{1}{\sqrt{2}} \sin^{-1} \left( \frac{4t-3}{3} \right) + c$$

$$16. 3\sqrt{x^2 + 9} - \log|x + \sqrt{x^2 + 9}| + c$$

**17.**  $\frac{x-1}{2} \sqrt{5-2x+x^2} + 2\log|x-1+\sqrt{5-2x+x^2}|+c$

**18.**  $\frac{1}{4}\{\log|x^2-1|-\log|x^2+1|\}+c$       **19.**  $\frac{1}{4}\left\{\log\left|\frac{1+x}{1-x}\right|\right\}-\frac{1}{2}\tan^{-1}x+c$

**20.**  $\frac{x-a}{2}\sqrt{2ax-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right)+c$     **21.**  $\frac{x\sin^{-1}x}{\sqrt{1-x^2}}+\log|\sqrt{1-x^2}|$

**22.**  $-\left(\frac{1}{2}\sin 2x+\sin x\right)+c$       **23.**  $\tan x - \cot x - 3x + c$

**24.**  $\frac{2}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}}+c$       **25.**  $2\sin x+x+c$

**26.**  $\frac{1}{2}\sec^{-1}(x^2)+c$       **27.**  $\frac{26}{3}$

**28.**  $e^2-1$       **29.**  $\tan^{-1}e-\frac{\pi}{4}$       **30.**  $\frac{\log m}{m^2-1}$       **31.**  $\pi$

**32.**  $\sqrt{2}-1$       **33.**  $\frac{\pi}{3}$       **34.**  $\frac{\sqrt{2}}{2}\tan^{-1}\frac{\sqrt{2}}{3}$

**35.**  $\frac{1}{7}\log\left|\frac{x-2}{x+2}\right|+\frac{\sqrt{3}}{7}\tan^{-1}\frac{x}{\sqrt{3}}+c$

**36.**  $\frac{1}{a^2-b^2}\left\{a\tan^{-1}\frac{x}{a}-b\tan^{-1}\frac{x}{b}\right\}+c$     **37.**  $\pi$

**38.**  $\log\left|\frac{\sqrt{x-3}}{(x-1)^{\frac{1}{6}}(x+2)^{\frac{1}{3}}}\right|+c$       **39.**  $xe^{\tan^{-1}x}+c$

**40.**  $a\left[\frac{x}{a}\tan^{-1}\sqrt{\frac{x}{a}}-\sqrt{\frac{x}{a}}+\tan^{-1}\sqrt{\frac{x}{a}}\right]+c$     **41.**  $\frac{3}{2}$

**42.**  $\frac{e^{-3x}}{24} [\sin 3x - \cos 3x] + \frac{3e^{-3x}}{40} [\sin x - 3\cos x] + c$

**43.**  $\frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{\tan x - 1}{\sqrt{2 \tan x}} \right) + \frac{1}{2\sqrt{2}} \log \left| \frac{\tan x - \sqrt{2 \tan x} + 1}{\tan x + \sqrt{2 \tan x} + 1} \right| + c$

**44.**  $\frac{1}{4} \left( \frac{a^2 + b^2}{a^3 b^3} \right)$     **45.**  $\frac{3}{8} \log 3$     **46.**  $\frac{1}{2} \log \frac{1}{2}$     **47.**  $\frac{1}{4} \log \frac{1}{2}$

**48.** A    **49.** C    **50.** A    **51.** C

**52.** D    **53.** C    **54.** D    **55.** D

**56.** D    **57.** A    **58.** D    **59.**  $e^{-1}$

**60.**  $\frac{e^x}{x+4} + c$     **61.**  $\frac{1}{2}$     **62.**  $\frac{-1}{2\sqrt{3}} \tan^{-1} \left( \frac{2\cos x}{\sqrt{3}} \right) + c$     **63.** 0

### 8.3 EXERCISE

**1.**  $\frac{1}{2}$  sq.units    **2.**  $\frac{4}{3} p^2$  sq. units    **3.** 10 sq.units    **4.**  $\frac{16}{3}$  sq.units

**5.**  $\frac{27}{2}$  sq.units    **6.**  $\frac{9}{2}$  sq. units    **7.**  $\frac{32}{3}$  sq. units    **8.**  $2\pi$  sq.units

**9.**  $\frac{4}{3}$  sq.units    **10.** 96 sq.units    **11.**  $\frac{16}{3}$  sq.units    **12.**  $\frac{a^2}{4}$  sq. units

**13.**  $\frac{1}{6}$  sq. units    **14.**  $\frac{9}{2}$  sq. units    **15.** 9 sq.units    **16.**  $2 \left[ -\frac{8}{3} \right]$  sq.units

**17.** 4 sq.units    **18.**  $\frac{15}{2}$  sq. units    **19.**  $\frac{4}{3} (\sqrt{3} + 2) a^2$  sq. units

**20.** 6 sq.units    **21.**  $\frac{15}{2}$  sq. units    **22.** 8 sq.units    **23.** 15 sq.units

**24.** C    **25.** D    **26.** A    **27.** B

**28.** A**29.** A**30.** D**31.** A**32.** B**33.** A**34.** C

### 9.3 EXERCISE

$$\mathbf{1.} \quad 2^{-x} - 2^{-y} = k$$

$$\mathbf{2.} \quad \frac{d^2y}{dx^2} = 0$$

$$\mathbf{3.} \quad \frac{e^6 + 9}{2}$$

$$\mathbf{4.} \quad y(x^2 - 1) = \frac{1}{2} \log \left( \left| \frac{x-1}{x+1} \right| \right) + k$$

$$\mathbf{5.} \quad y = c \cdot e^{x-x^2}$$

$$\mathbf{6.} \quad (a+m)y = e^{mx} + ce^{-ax}$$

$$\mathbf{7.} \quad (x-c)e^{x+y} + 1 = 0$$

$$\mathbf{8.} \quad y = kx e^{\frac{-x^2}{2}}$$

$$\mathbf{9.} \quad y = \tan \left( x + \frac{x^2}{2} \right)$$

$$\mathbf{10.} \quad x = y(y^2 + c)$$

$$\mathbf{11.} \quad \frac{1}{3}$$

$$\mathbf{13.} \quad (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$$

$$\mathbf{14.} \quad (x^2 - y^2) \frac{dy}{dx} - 2xy = 0$$

$$\mathbf{15.} \quad y = \frac{4x^3}{3(1+x^2)}$$

$$\mathbf{16.} \quad \tan^{-1} \left( \frac{y}{x} \right) = \log|x| + c$$

$$\mathbf{17.} \quad 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + c$$

$$\mathbf{18.} \quad \tan^{-1} \left( \frac{x}{y} \right) + \log y = c$$

$$\mathbf{19.} \quad x + y = k e^{x-y}$$

$$\mathbf{20.} \quad x^2(y+3)^3 = e^{y+2}$$

$$\mathbf{21.} \quad y \sin x = \frac{-\cos 2x}{2} + \frac{3}{2}$$

$$\mathbf{22.} \quad xy y' + x(\dot{y})^2 - \dot{y}y = 0$$

$$\mathbf{23.} \quad \frac{1}{2} (\tan^{-1} x)^2 + \log(1+y^2) = c$$

$$\mathbf{24.} \quad (x-1) + (y-2) \frac{dy}{dx} = 0$$

$$\mathbf{25.} \quad y = -\cos x + \frac{2\sin x}{x} + \frac{2\cos x}{x^2} + \frac{x \log x}{3} - \frac{x}{9} + cx^{-2}$$

**26.**  $x(\sin y + \cos y) = \sin y + c e^{-y}$

**27.**  $\log \left| 1 + \tan \left( \frac{x+y}{2} \right) \right| = x + c$

**28.**  $y = -\left[ \frac{3\sin 2x + 2\cos 2x}{13} \right] + ce^{3x}$

**29.**  $2(x^2 - y^2) = 3x$

**30.**  $(y-1)(x+1) + 2x = 0$

**31.**  $k e^{2x}(1-x+y) = 1+x-y$

**32.**  $xy = 1$

**33.**  $\log \left( \frac{x}{y} \right) = cx$

**34.** D

**35.** C

**36.** A

**37.** C

**38.** B

**39.** C

**40.** C

**41.** D

**42.** A

**43.** C

**44.** D

**45.** B

**46.** B

**47.** C

**48.** C

**49.** D

**50.** A

**51.** A

**52.** B

**53.** B

**54.** B

**55.** B

**56.** C

**57.** B

**58.** A

**59.** A

**60.** C

**61.** C

**62.** D

**63.** C

**64.** C

**65.** A

**66.** D

**67.** D

**68.** C

**69.** C

**70.** A

**71.** A

**72.** A

**73.** C

**74.** B

**75.** A

**76.** (i) not defined

(ii) not defined

(iii) 3

(iv)  $\frac{dy}{dx} + py = Q$

(v)  $xe^{\int p_1 dy} = \int (Q_1 \times e^{\int p_1 dy}) dy + c$

(vi)  $y = \frac{x^2}{4} + cx^{-2}$

(vii)  $3y(1+x^2) = 4x^3 + c$

(viii)  $xy = Ae^{-y}$

(ix)  $y = ce^{-x} + \frac{\sin x}{2} - \frac{\cos x}{2}$

(x)  $x = c \sec y$

(xi)  $\frac{e^x}{x}$

**77.** (i) True

(ii) True

(iii) True

(iv) True

(v) False

(vi) False

(vii) True

(viii) True

(ix) True

(x) True

(xi) True

### 10.3 EXERCISE

1.  $\frac{1}{3}(2\hat{i} + \hat{j} + 2\hat{k})$     2. (i)  $\frac{1}{3}(2\hat{i} + \hat{j} - 2\hat{k})$     (ii)  $\frac{1}{\sqrt{37}}(\hat{j} + 6\hat{k})$

3.  $\frac{1}{7}(-2\hat{i} + 3\hat{j} - 6\hat{k})$     4.  $\vec{c} = \frac{3\bar{b} - \bar{a}}{2}$     5.  $k = -2$     6.  $\pm 2(\hat{i} + \hat{j} + \hat{k})$

7.  $\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}; 4\hat{i}, 6\hat{j}, -12\hat{k}$     8.  $-2\hat{i} + 4\hat{j} + 4\hat{k}$     9.  $\cos^{-1}\left(\frac{1}{\sqrt{156}}\right)$

10. Area of the parallelograms formed by taking any two sides represented by  $\bar{a}, \bar{b}$  and  $\bar{c}$  as adjacent are equal

11.  $\frac{2}{\sqrt{7}}$

12.  $\sqrt{21}$

13.  $\frac{\sqrt{274}}{2}$

16.  $\hat{n} = \frac{\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}$

17.  $\frac{\sqrt{62}}{2}$

18.  $\frac{1}{3}(5\vec{i} + 2\vec{j} + 2\vec{k})$

19. C

20. D

21. C

22. B

23. D

24. A

25. D

26. D

27. D

28. A

29. C

30. A

31. C

32. C

33. B

34. If  $\bar{a}$  and  $\bar{b}$  are equal vectors

35. 0

36.  $\frac{\pi}{4}$

37.  $k \in ]-1, 1[ \setminus \{0\}$

38.  $|\vec{a}|^2 |\vec{b}|^2$

39. 3

40.  $\vec{a}$

41. True    42. True

43. True

44. False

45. False

### 11.3 EXERCISE

1.  $5\hat{i} + 5\sqrt{2}\hat{j} + 5\hat{k}$     2.  $(x-1)\hat{i} + (y+2)\hat{j} + (z-3)\hat{k} = \lambda(3\hat{j} - 2\hat{j} + 6\hat{k})$

3.  $(-1, -1, -1)$

**4.**  $\cos^{-1}\left(\frac{19}{21}\right)$

**7.**  $x + y + 2z = 19$  **8.**  $x + y + z = 9$

**9.**  $3x - 2y + 6z - 27 = 0$

**10.**  $21x + 9y - 3z - 51 = 0$

**11.**  $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$  and  $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$

**12.**  $60^\circ$

**14.**  $ax + by + cz = a^2 + b^2 + c^2$

**14.**  $(1, 1)$

**15.**  $15^\circ$  or  $75^\circ$

**16.**  $(2, 6, -2)$   $3\sqrt{5}$

**17.** 7

**18.**  $\sqrt{6}$

**19.**  $(x-3)\hat{i} + y\hat{j} + (z-1)\hat{k} = \lambda(-2\hat{i} + \hat{j} + 3\hat{k})$

**20.**  $18x + 17y + 4z = 49$  **21.** 14

**22.**  $51x + 15y - 50z + 173 = 0$

**24.**  $4x + 2y - 4z - 6 = 0$  and  $-2x + 4y + 4z - 6 = 0$

**26.**  $3\hat{i} + 8\hat{j} + 3\hat{k}, -3\hat{i} - 7\hat{j} + 6\hat{k}$

**29.** D

**30.** D

**31.** A

**32.** D

**33.** D

**34.** A

**35.** D

**36.** C

**37.**  $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$

**38.**  $\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$

**39.**  $(x-5)\hat{i} + (y+4)\hat{j} + (z-6)\hat{k} = \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

**40.**  $(x-3)\hat{i} + (y-4)\hat{j} + (z+7)\hat{k} = \lambda(-2\hat{i} - 5\hat{j} + 13\hat{k})$

**41.**  $x + y - z = 2$

**42.** True

**43.** True

**44.** False

**45.** False

**46.** True

**47.** True

**48.** False

**49.** True

### 12.3 EXERCISE

**1.** 42

**2.** 4

**3.** 47

**4.**  $-30$

**5.** 196

**6.** 43

**7.** 21

**8.** 47

**9.** Minimum value = 3

**10.** Maximum = 9, minimum =  $3\frac{1}{7}$

- 11.** Maximise  $Z = 50x + 60y$ , subject to:

$$2x + y \leq 20, x + 2y \leq 12, x + 3y \leq 15, x \geq 0, y \geq 0$$

- 12.** Minimise  $Z = 400x + 200y$ , subject to:

$$5x + 2y \geq 30$$

$$2x + y \leq 15$$

$$x \leq y, x \geq 0, y \geq 0$$

- 13.** Maximise  $Z = 100x + 170y$  subject to :

$$3x + 2y \leq 3600, x + 4y \leq 1800, x \geq 0, y \geq 0$$

- 14.** Maximise  $Z = 200x + 120y$  subject to :

$$x + y \leq 300, 3x + y \leq 600, y \leq x + 100, x \geq 0, y \geq 0$$

- 15.** Maximise  $Z = x + y$ , subject to

$$2x + 3y \leq 120, 8x + 5y \leq 400, x \geq 0, y \geq 0$$

- 16.** Type A : 6, Type B : 3; Maximum profit = Rs. 480

**17.** 2571.43

**18.** 138600

- 19.** 150 sweaters of each type and maximum profit = Rs 48,000

**20.**  $54\frac{2}{7}$  km.

**21.**  $3\frac{10}{11}$

- 22.** Model X : 25, Model Y : 30 and maximum profit = Rs 40,000

- 23.** Tablet X : 1, Tablet Y : 6      **24.** Factory I : 80 days, Factory II : 60 days

- 25.** Maximum : 12, Minimum does not exist

**26.** B

**27.** B

**28.** A

**29.** D

**30.** C

**31.** D

**32.** D

**33.** A

**34.** B

**35.** Linear constraints

**36.** Linear

**37.** Unbounded

**38.** Maximum

**39.** Bounded

**40.** Intersection

**41.** Convex

**42.** True

**43.** False

**44.** False

**45.** True

**13.3 EXERCISE**

**1.** Independent    **2.** not independent    **3.** 1.1    **4.**  $\frac{25}{56}$

**5.**  $P(E) = \frac{1}{12}$ ,  $P(F) : \frac{5}{18}$ ,  $P(G) = \frac{7}{36}$ , no pair is independent

**7.** (i)  $\frac{3}{4}$ , (ii)  $\frac{1}{2}$ , (iii)  $\frac{1}{4}$ , (iv)  $\frac{5}{8}$     **8.**  $\frac{3}{4}, \frac{3}{10}$

**9.** (i)  $E_1$  and  $E_2$  occur

(ii)  $E_1$  does not occur, but  $E_2$  occurs

(iii) Either  $E_1$  or  $E_2$ , or both  $E_1$  and  $E_2$  occurs

(iv) Either  $E_1$  or  $E_2$  occurs, but not both

**10.** (i)  $\frac{1}{3}$ , (ii)  $\frac{23}{18}$     **12.**  $\frac{\sqrt{3}}{2}$     **13.** Rs 0.50    **14.**  $\frac{1}{10}$

**15.** Expectation = Rs 0.65    **16.**  $\frac{85}{153}$     **17.**  $\frac{7}{15}$

**18.**  $\frac{5}{9}$     **19.**  $\frac{1}{270725}$     **20.**  $\frac{5}{16}$     **21.**  $\frac{7}{128}$

**22.**  $\frac{4547}{8192}$     **23.**  $1 - \left(\frac{9}{10}\right)^8$     **24.** (i) .1118    (ii) .4475

**25.** (i)  $\frac{8}{15}$ , (ii)  $\frac{14}{15}, \frac{1}{15}$ , (iii) 1    **26.** 0.7 (approx.)    **27.** 0.18

**28.**  $\frac{1}{2}$     **29.**

X	0	1	2
P(X)	.54	.42	.04

**31.** (i)  $\left(\frac{49}{50}\right)^{10}$     (ii)  $\frac{45(49)^8}{(50)^{10}}$     (iii)  $\frac{59(49)^9}{(50)^{10}}$

**32.**  $\frac{1}{3}$

**33.**  $\frac{9}{44}$

**34.**  $\frac{p-1}{n-1}$

**35.**

X	1	2	3	4	5	6
P(X)	36	36	36	36	36	36

**36.**  $p = \frac{1}{2}$

**37.**  $\frac{665}{324}$

**38.**  $\frac{775}{776}$

**39.** not independent

**41.** (i)  $\frac{7}{18}$ , (ii)  $\frac{11}{18}$

**42.** (i)  $\frac{2}{11}$ , (ii)  $\frac{9}{11}$

**43.** (i) 0.49, (ii) 0.65, (iii) .314

**44.**  $\frac{7}{11}$

**45.**  $\frac{11}{21}$

**46.**  $\frac{1}{3}$

**47.**  $\frac{110}{221}$

**48.**  $\frac{5}{11}$

**49.** (i)  $\frac{1}{50}$ , (ii) 5.2, (iii) 1.7 (approx.)

**50.** (i) 3, (ii) 19.05

**51.** (i) 4.32, (ii) 61.9, (iii)  $\frac{15}{22}$

**52.** 10

**53.** Mean =  $\frac{2}{13}$ , S.D. = 0.377

**54.**  $\frac{1}{2}$

**55.** Mean = 6, Variance = 3

**56.** C

**57.** A

**58.** D

**59.** C

**60.** C

**61.** D

**62.** B

**63.** D

**64.** C

**65.** D

**66.** D

**67.** D

**68.** C

**69.** D

**70.** D

**71.** D

**72.** C

**73.** C

**74.** C

**75.** B

**76.** B

**77.** D

**78.** C

**79.** A

**80.** D

**81.** B

**82.** C

**83.** C

**84.** A

**85.** B

**86.** A

**87.** C

**88.** D

**89.** D

**90.** A

**91.** B

**92.** D**93.** D**94.** False**95.** True**96.** False**97.** False**98.** True**99.** True**100.** True**101.** True**102.** False**103.** True**104.**  $\frac{1}{3}$ **105.**  $\frac{10}{9}$ **106.**  $\frac{1}{10}$ **107.**  $\sum p_i x_i^2 - (\sum p_i x_i)^2$ **108.** independent