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## INTRODUCTION

Do you remember numbers? Let us solve some problems.

## 1. Fill in the following blanks.

(a) The place value of 5 in 37572 is \_\_\_\_\_

(b) 8 occurs at \_\_\_\_\_ place in 105876.

(c) Place value of 4 in 42160 is \_\_\_\_\_

(d) 5 occupies the \_\_\_\_\_ place in 37652.

(e) The face value of 7 in 4709606 is \_\_\_\_\_

(f)  $3 \times 100000 + 5 \times 1000 + 7 \times 10 + 8 \times 1 =$  \_\_\_\_\_

(g)  $200000 + 4000 + 800 + 6 =$  \_\_\_\_\_

## 2. Find the product of the place value and face value of 5 in 76085432.

## 3. Find the product of the largest 4-digit number and the smallest 4-digit number. Write the product in expanded form also.

## 4. Write all the possible 3-digit numbers using the digits 7, 5, 1.

**(Repetition not allowed)**

## 5. Write all the possible 3-digit numbers using the digits 4, 0, 6.

**(Repetition not allowed)**

## 6. Write the following numbers in Indian System of Numeration.

(a) 8751432

(b) 60002

(c) 491603

(d) 632245687

## 7. Write the following numbers in International System of Numeration.

(a) 5737802

(b) 411809

(c) 33246951

(d) 898576449

## 8. Write the numerals for the following:

(a) Thirty two million four thousand three hundred and twenty nine.

(b) Thirty nine crore forty eight lakh nine thousand and eighty eight.

## 9. How many lakhs make 6 millions?

## 10. How many millions make 17 crores?

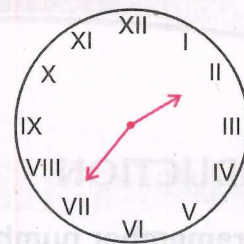


## ROMAN NUMERALS

Have you ever seen a clock of this type?



See! In place of numerals 1 to 12, symbols like I, II, III, IV, ... are shown here.



These symbols are called **Roman Numerals**.

Now observe these **Hindu Arabic Numerals** and their corresponding **Roman Numerals**.

<b>Hindu Arabic Numerals</b>	1	5	10	50	100	500	1000
<b>Roman Numerals</b>	I	V	X	L	C	D	M

The rules for this system of numeration are given below:

- **Rule 1 –** If a symbol is repeated, its value is added as many times as it occurs.  
For example:  $II = 1 + 1 = 2$   
 $XXX = 10 + 10 + 10 = 30$
- **Rule 2 –** A symbol is not repeated more than three times but the symbols V, L and D are never repeated.
- **Rule 3 –** If a symbol of smaller value is written to the right of a symbol of greater value, its value gets added to the value of the greater symbol.

For example:  $VI = 5 + 1$

$$= 6$$

$$LXV = 50 + 10 + 5$$

$$= 65$$

- **Rule 4 –** If a symbol of smaller value is written to the left of a symbol of greater value, its value is subtracted from the symbol of the greater value.

For example:  $IV = 5 - 1 = 4$

$$XL = 50 - 10 = 40$$

$$XC = 100 - 10 = 90$$

- **Rule 5 –** The symbols V, L and D are never written to the left of a symbol of greater value, i.e. V, L, D are never subtracted.

Observe the Roman Numerals corresponding to some Hindu Arabic Numerals.

1 = I	10 = X
2 = II	20 = XX
3 = III	30 = XXX
4 = IV	40 = XL
5 = V	50 = L
6 = VI	60 = LX
7 = VII	70 = LXX
8 = VIII	80 = LXXX
9 = IX	90 = XC
10 = X	100 = C

Let us study some examples.

**Example 1:** Write the Roman Numerals corresponding to the following Hindu Arabic Numerals.

- (a) 19      (b) 56      (c) 44      (d) 98      (e) 78

**Solution:**

(a)  $19 = 10 + 9$   
 $= XIX$

(b)  $56 = 50 + 6$   
 $= LVI$

(c)  $44 = 40 + 4$   
 $= XLIV$

(d)  $98 = 90 + 8$   
 $= XCVIII$

(e)  $78 = 70 + 8$   
 $= (50 + 10 + 10) + 8$   
 $= LXXVIII$

**Example 2:** Convert the following into Hindu Arabic Numerals.

- (a) LXXIX      (b) XLIX      (c) XCVII      (d) XCI

**Solution:**

(a)  $LXXIX = 50 + 10 + 10 + 9$   
 $= 79$

(b)  $XLIX = 40 + 9$   
 $= 49$

(c)  $XCVII = 90 + 7$   
 $= 97$

(d)  $XCI = 90 + 1$   
 $= 91$

## Worksheet 1

1. Write the Roman Numeral for each of the following:

- (a) 33                      (b) 500                      (c) 48                      (d) 76                      (e) 95  
(f) 41                      (g) 87                      (h) 66                      (i) 19                      (j) 1000

2. Convert the following into Hindu Arabic Numerals.

- (a) XXVI                      (b) LXXVII                      (c) XCI                      (d) LXXXV                      (e) D  
(f) XCIX                      (g) XCVII                      (h) LV                      (i) XLI                      (j) XXIX

3. Solve and write the results in Roman Numerals.

- (a)  $32 + 67$                       (b)  $216 - 174$   
(c)  $12 \times 7$                       (d)  $3645 \div 45$

4. Which of the following is meaningless?

- (a) VII                      (b) XLI                      (c) LIV                      (d) IC                      (e) LIL  
(f) IVC                      (g) XCI                      (h) VL

5. Match the following:

DXLV	908
MMX	591
CMVIII	545
CCIII	2010
DXCI	203

6. Write the following in Roman Numerals.

- (a) Year in which India got Independence.  
(b) Year in which India became Republic.  
(c) Year in which you were born.  
(d) Present year.

## WHOLE NUMBERS AND THEIR REPRESENTATION ON A NUMBER LINE



How many legs does a spider have?

A spider has 8 legs.



How many paise are there in one rupee?

There are 100 paise in one rupee.



So we have used the numbers 1, 2, 3, 4, ..... for answering these questions.

Numbers 1, 2, 3, 4, ....., which we use for counting, form the system of **Natural Numbers** (Counting numbers).

### Remember

- The smallest natural number is 1.
- We cannot find the greatest natural number.

Look at the following picture. What is the number of boys in this group?



The number of boys in this group is zero (0).

Natural numbers along with zero form the system of **Whole Numbers**.

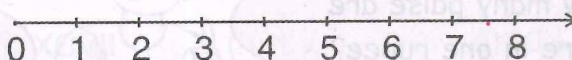
### Remember

- The smallest whole number is 0.
- We cannot find the greatest whole number.

### For the teacher:

Explain to the students that these numbers are equidistant on the number line.

Now look at the whole numbers given on a number line.



## SUCCESSOR AND PREDECESSOR

One more than any whole number is called the **successor** of that whole number.

For example: 51 is the successor of 50

10000 is the successor of 9999

Number to the Right

One less than any whole number is called the **predecessor** of that whole number.

For example: 61 is the predecessor of 62

99999 is the predecessor of 100000

Number to the Left

Let us take up some examples.

**Example 3:** Write the greatest 4-digit number using the digits 5, 0, 2. (**digits may repeat**)

**Solution:** Any 4-digit number occupies four places, i.e. thousands, hundreds, tens and ones. Since 5 is the largest number here, it will occupy most of the places in the required number and rest of the numbers will occur only once and that too in descending order. So, the required number will be,

Th H T O

5	5	2	0
---	---	---	---

**Example 4:** Rearrange the digits of 72094186 to form the smallest 8-digit number.

**Solution:** We write the digits in ascending order—

0, 1, 2, 4, 6, 7, 8, 9

Since we cannot start a number with zero, we start the number with 1. So the required number is—

1, 02, 46, 789

## Worksheet 2

1. Complete the statements by filling in the blanks.

- (a) The smallest whole number is \_\_\_\_\_.
- (b) There is \_\_\_\_\_ largest whole number.
- (c) In whole numbers, \_\_\_\_\_ has no predecessor.
- (d) The predecessor of the smallest 5-digit number has \_\_\_\_\_ digits.
- (e) The successor of the greatest 5-digit number is \_\_\_\_\_.
- (f) The smallest 7-digit number ending in 5 is \_\_\_\_\_.
- (g) 387 is to the \_\_\_\_\_ of 388 on the number line.
- (h) 4397 is to the \_\_\_\_\_ of 4396 on the number line.

2. Write the successor of the following:

- (a) 45638      (b) 10009      (c) 220209      (d) 4226372

3. Write the predecessor of the following:

- (a) 33801      (b) 100000      (c) 6698979      (d) 80115670

4. Find the next three successors of 647999.

5. Find the three immediate predecessors of 552002.

6. Compare the following numbers:

- (a) 729  279      (b) 10899  10799  
(c) 9785  7835      (d) 135629  136529

7. Arrange the following in ascending order.

43, 287, 15769, 833, 49538, 34, 798665

8. Arrange the following in descending order.

3951, 1024, 977, 422596, 38675, 560832, 67.

9. Form the greatest 7-digit number using the digits 3, 8, 9.

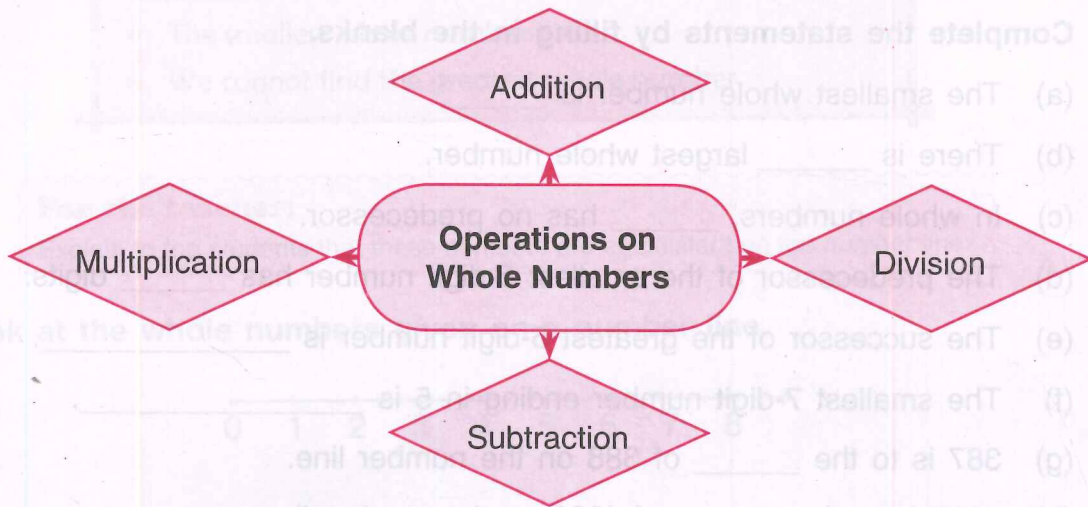
(digits may repeat)

10. Write the smallest 6-digit number using the digits 4, 5, 0.

(digits may repeat)



# OPERATIONS ON WHOLE NUMBERS



Let us take up the properties of each and every operation one by one.

## A. ADDITION OF WHOLE NUMBERS

### Properties of Addition

**Property-1:** The sum of two whole numbers is again a whole number.

e.g.  $3 + 8 = 11$

**Property-2:** The sum remains the same even after changing the order of addends.

e.g.  $23 + 18 = 18 + 23$

**Property-3:** The sum remains the same, when the order or the grouping of three or more addends is changed.

e.g.  $11 + (18 + 25) = (11 + 18) + 28$

**Property-4:** When a number is added to zero or zero is added to the number, sum is the number itself.

e.g.  $7 + 0 = 0 + 7 = 7$

Let us take up an example to see that the sum remains same even if the order of the addends is changed.

**Example 5:** Add 469, 35, 31, 5 in 2 different ways.

**Solution:**

$$\begin{array}{r} 469 + 35 + 5 + 31 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 504 \quad + \quad 36 \\ \swarrow \quad \searrow \\ 540 \end{array} \qquad \begin{array}{r} 469 + 31 + 35 + 5 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 500 \quad + \quad 40 \\ \swarrow \quad \searrow \\ 540 \end{array}$$

(By interchanging the order of the addends.)

## B. SUBTRACTION OF WHOLE NUMBERS

### Properties of Subtraction

- Property-1:** The difference between two whole numbers may or may not be a whole number. e.g.  $5 - 4 = 1$  (is a whole number). But  $4 - 5 = -1$  is not a whole number.
- Property-2:** The difference between two same whole numbers is always zero. e.g.  $5 - 5 = 0$
- Property-3:** For any three whole numbers, say 6, 4, 2  $(6 - 4) - 2$  is not equal to  $6 - (4 - 2)$ .
- Property-4:** When zero is subtracted from a whole number, the difference is the number itself. e.g.  $5 - 0 = 5$
- Property-5:** If 8, 5, 3 are whole numbers, such that  $8 - 5 = 3$  then  $5 + 3 = 8$

Let us study an example based on Property-5.

**Example 6:** Subtract 40 from 96

**Solution:**

$$96 - 40 = 56$$

or

$$96 = 56 + 40$$


### Worksheet 3

1. Fill in the blanks to make the following statements true.

- (a)  $1794 + 624 = 624 + \underline{\hspace{2cm}}$
- (b)  $(287 + 163) + 800 = 287 + (\underline{\hspace{2cm}} + 800)$
- (c)  $432 + \underline{\hspace{2cm}} = 111 + 432$
- (d)  $97 + 561 = \underline{\hspace{2cm}} + 97$
- (e)  $(200 + 1020) + 3303 = \underline{\hspace{2cm}} + (200 + \underline{\hspace{2cm}})$
- (f)  $0 + 268 = \underline{\hspace{2cm}}$
- (g)  $469 - 0 = \underline{\hspace{2cm}}$
- (h)  $1238 - \underline{\hspace{2cm}} = 1238$
- (i)  $29487 + \underline{\hspace{2cm}} = 29487$

2. Replace (\*) with the appropriate digit.

(a) 
$$\begin{array}{r} 29422 \\ - 68*5 \\ \hline 2*547 \end{array}$$

(b) 
$$\begin{array}{r} 478*5 \\ + 3*334 \\ \hline 84*39 \end{array}$$

(c) 
$$\begin{array}{r} 239*8 \\ + 1*980* \\ \hline 143*69 \end{array}$$

(d) 
$$\begin{array}{r} 80019 \\ - *43** \\ \hline 25*03 \end{array}$$

3. Add 718662 to 360895. Now add 360895 to 718662. Are the two results same?

4. Add the following numbers by rearranging them: **(Use property here)**

(a)  $786 + 342 + 214$

(d)  $67 + 1376 + 624 + 933$

(b)  $479 + 2000 + 21$

(e)  $637 + 908 + 363$

(c)  $225 + 725 + 275 + 275$

(f)  $2062 + 547 + 938 + 353$

5. Subtract the following and check your answer by corresponding addition.

(a)  $29435 - 17005$

(b)  $100000 - 62581$

(c)  $75691 - 45512$

(d)  $77426 - 71236$

6. In a school, the number of students is 5637. If 142 students took admission during that year, find the total number of students in the school.

7. The price of a car is ₹ 3,76,866. If it is increased by ₹ 42,049, find the new price of the car.

8. A club organises a trip to the Disney World. The cost of the whole package is ₹ 1,83,420. The club gives a discount of ₹ 47,632. What is the cost of the package after the discount?

9. Rahul deposited ₹ 57,630 in the bank. After a week, he withdrew ₹ 19,211. What is the current balance in Rahul's account?

10. A garment factory produces 33000 trousers every year. Out of these, 12309 are for men and 9538 are for women. Find the number of trousers produced for children.

## C. MULTIPLICATION OF WHOLE NUMBERS

### Properties of Multiplication

**Property-1:** If two whole numbers are multiplied in either order, the product remains the same.

e.g.  $3 \times 8 = 8 \times 3 = 24$

**Property-2:** If three numbers are multiplied in any grouping or order, the product remains the same.

e.g.  $2 \times (5 \times 7) = (2 \times 5) \times 7 = (2 \times 7) \times 5 = 70$

**Property-3:** The product of a whole number and 1 is the number itself.

e.g.  $1 \times 5 = 5 \times 1 = 5$

**Property-4:** The product of any whole number and zero is zero.

e.g.  $2 \times 0 = 0 \times 2 = 0$

### Worksheet 4

1. Use the properties of multiplication and fill in the following blanks.

(a)  $0 \times 489 = \underline{\hspace{2cm}}$

(b)  $1 \times 741 = \underline{\hspace{2cm}}$

(c)  $27 \times 635 = 635 \times \underline{\hspace{2cm}}$

(d)  $(242 \times 197) \times 581 = 242 \times (197 \times \underline{\hspace{2cm}})$

(e)  $479 \times \underline{\hspace{2cm}} = 479$

(f)  $\underline{\hspace{2cm}} \times 831 = 0$

(g)  $162 \times 0 \times 1025 = \underline{\hspace{2cm}}$

2. If the cost of one burger is ₹ 50.50, what will be the cost of 25 such burgers?

3. In a library, there are 27 book shelves. If there are 479 books on each book shelf, find the total number of books in the library.

4. A store has 432 dresses for girls. If the cost of each dress is ₹ 583.50, find the cost of all dresses.

### MORE ABOUT MULTIPLICATION PROPERTIES

Consider the numbers 3, 4 and 5.

Let us add 3 and 4 and multiply the sum by 5

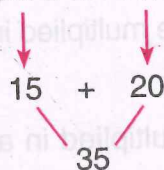
$$(3 + 4) \times 5$$

$$\downarrow$$
$$7 \times 5$$

$$= 35$$

Now multiply 3 and 4 separately by 5 and then add the products.

$$3 \times 5 + 4 \times 5$$



In both the cases, we get 35. So, we can say that—

$$(3 + 4) \times 5 = 3 \times 5 + 4 \times 5$$

Similarly,  $(7 - 3) \times 5 = 7 \times 5 - 3 \times 5$

This is known as **Distributive Property of Multiplication**. It is useful for multiplying large numbers.

**Example 7:** Multiply  $172 \times 97$

**Solution:** We know that  $97 = (100 - 3)$   
 $172 \times (100 - 3)$

or  $172 \times (100 - 3) = 172 \times 100 - 172 \times 3$   
 $= 17200 - 516$   
 $= 16684$

**Example 8:** Solve  $569 \times 45 + 569 \times 55$

**Solution:**  $569 \times (45 + 55)$  ← Taking out 569 as common factor from both the products.

$= 569 \times 100$  ← Multiplying by 100 orally.

$= 56900$

**Example 9:** Solve  $361 \times 162 - 361 \times 60 - 2 \times 361$

**Solution:**  $361 \times (162 - 60 - 2)$  ← Taking out 361 as common factor and putting rest of the terms in a bracket.

$= 361 \times (162 - 62)$

$= 361 \times 100$

$= 36100$

### Remember

By rearranging the order of the numbers, multiplication becomes easy. We try to combine the numbers that produce maximum number of zeroes after the multiplication.

## Worksheet 5

1. Fill in the following blanks by using different properties of multiplication.

(a)  $52 \times (63 + 37) = (52 \times \underline{\quad}) + (\underline{\quad} \times 37)$

(b)  $297 \times (\underline{\quad} + 43) = (297 \times 88) + (297 \times \underline{\quad})$

(c)  $\underline{\quad} \times (84 + 16) = 36 \times 84 + 36 \times \underline{\quad}$

(d)  $218 \times 94 = (218 \times \underline{\quad}) - (218 \times 6)$

(e)  $778 \times 994 = (778 \times 1000) - (778 \times \underline{\quad}) - 778.$

2. Rearrange the numbers and then multiply them.

(a)  $125 \times 488 \times 8$

(b)  $625 \times 25 \times 20 \times 4$

(c)  $16 \times 125 \times 8 \times 625$

(d)  $20 \times 1975 \times 5$

(e)  $8 \times 25 \times 125 \times 40$

(f)  $200 \times 625 \times 16 \times 50$

3. Find the product by using distributive property.

(a)  $241 \times 107$

(b)  $685 \times 94$

(c)  $439 \times 995$

(d)  $1009 \times 1392$

(e)  $98 \times 553$

(f)  $999 \times 399$

4. Find the value by using distributive property.

(a)  $1562 \times 62 + 1562 \times 38$

(b)  $638 \times 176 - 638 \times 75 - 638$

(c)  $85 \times 15 + 15 \times 15$

(d)  $688 \times 10 \times 437 - 6880 \times 337$

(e)  $125 \times 8 \times 883 + 117 \times 25 \times 40$

(f)  $750 \times 17 + 750 \times 38 + 27 \times 750 + 18 \times 750$

5. Rohan buys 12 computers and 12 printers. If the cost of one computer and one printer is ₹ 56,233 and ₹ 7,867 respectively, find the total cost incurred by Rohan. (Use distributive property of multiplication.)

6. In a school, the monthly fee of a child is ₹ 497. If there are 2983 students in a school, find the total fee collected in a month.

(Use distributive property of multiplication.)

#### D. DIVISION OF WHOLE NUMBERS

**Property-1:** If two whole numbers are divided, their quotient may or may not be a whole number.

e.g.  $3 \div 6 = \frac{1}{2}$  but  $6 \div 3 = 2$

**Property-2:** A number divided by itself, gives the quotient as 1.

e.g.  $5 \div 5 = 1$ .

**Property-3:** A number divided by one gives the quotient as the number itself.

e.g.  $4 \div 1 = 4$

**Property-4:** A multiplication fact of two distinct and non-zero whole numbers gives two division facts.

e.g.  $4 \times 5 = 20$  and  $20 \div 5 = 4$ ,  $20 \div 4 = 5$

**Property-5:** Zero divided by any number gives the quotient as zero.

e.g.  $0 \div 3 = 0$

We also know

In division

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

Let us take up some examples.

**Example 10:** Find the least number that should be subtracted from 1000 so that 30 divides the difference exactly.

**Solution:** Divide 1000 by 30

$$\begin{array}{r} 30 \overline{) 1000} \quad 33 \\ \underline{90} \phantom{0} \\ 100 \\ \underline{90} \\ 10 \end{array}$$

$$1000 - 10 = 990$$

So, 10 should be subtracted from 1000 so that the difference, i.e. 990 is exactly divisible by 30.

**Example 11:** Find the least number that should be added to 1000 so that 35 divides the sum exactly.

**Solution:**

$$\begin{array}{r} 35 \overline{) 1000} \quad 24 \\ \underline{70} \\ 300 \\ \underline{280} \\ 20 \end{array}$$

The difference between divisor and remainder is  $35 - 20 = 15$

Therefore, 15 should be added to 1000 so that the sum 1015 is exactly divisible by 35.

### Worksheet 6

1. Divide and check your answer.

(a)  $2781 \div 35$

(b)  $49277 \div 511$

(c)  $7335 \div 122$

(d)  $64895 \div 247$

2. Find the least number that should be subtracted from 1000 so that 35 divides the difference exactly.

3. Find the least number that should be added to 2000 so that 45 divides the sum exactly.

4. Find the largest 5-digit number which is exactly divisible by 40.

5. In a parade, the soldiers are arranged in 14 rows. If the number of soldiers is 504, find the number of soldiers in each row.

6. In a dance class, 137 students got themselves enrolled. If the total fee collected is ₹ 3,56,200, find the fee paid by each student.

### ESTIMATION

Do you remember Rounding off numbers? Let us recall.

1. Round off the given numbers as directed.

(a) 48 (to the nearest ten)

(b) 3,285 (to the nearest thousand)

(c) 87,08,463 (to the nearest ten lakh)

(d) 4,53,73,043 (to the nearest crore)



2. Round off the given numbers as directed.

- (a) 3.84 (to the nearest ones)
- (b) 21.472 (to the nearest hundredth)
- (c) 1.53 (to the nearest tenth)

3. Round 4,25,163 to the nearest hundred, ten thousand and lakh.

## ESTIMATION OF OUTCOMES OF NUMBER SITUATIONS

Let us take some situations.

**Situation 1:** Rohan plans to give a treat to his eight friends in school on his birthday. His father gave him ₹ 500 for this. He decides to give a sandwich, pastry and fruit juice to these friends. One sandwich costs ₹ 20, one pastry costs ₹ 25 and one fruit juice costs ₹ 15. Rohan roughly calculates the amount he needed. This will be the sum of amount he spends on these three items.

**Situation 2:** On a particular day a businessman has to receive ₹ 5,38,485 and ₹ 2,19,560 from two different parties. He also has to pay a sum of ₹ 6,35,750 to someone on the same day. He quickly round off the numbers to the nearest lakh and then works out if he will be able to pay the money by evening. Will he be able to pay back the amount?

The estimation of outcomes of numbers is a reasonable guess of the actual value.

### Remember

- **Estimating** means approximating a quantity to the accuracy required.
- Estimation is done by rounding off the numbers involved and getting a quick and rough answer.

## ESTIMATION OF SUM OR DIFFERENCE

When we estimate sum or difference, we should have an idea of why we need to round off and therefore, the place to which the rounding is needed.

**Example 12:** Estimate  $4,356 + 13,849$

**Solution:** We shall round off the numbers to the nearest thousands.

13,849 is rounded off to 14,000

4,356 is rounded off to 4,000

$$\begin{aligned}\text{Estimated sum} &= 14,000 + 4,000 \\ &= 18,000\end{aligned}$$

**Example 13:** Estimate  $7,412 - 236$

**Solution:** Let us round off these numbers to the nearest thousands.

7,412 is rounded off to 7,000

236 is rounded off to 0

$$\begin{aligned}\text{Estimated difference} &= 7000 - 0 \\ &= 7000\end{aligned}$$

This is not a reasonable estimate. Why?

We need a closer estimate.

Let us round the numbers to the nearest hundreds.

7,412 is rounded off to 7,400

236 is rounded off to 200

$$\begin{aligned}\text{Estimated difference} &= 7,400 - 200 \\ &= 7,200\end{aligned}$$

This is a better and more meaningful estimate.

## Worksheet 7

**Estimate.**

1.  $215 + 436$

2.  $1,238 + 4,298$

3.  $15,409 + 3,288$

4.  $618 + 561 + 372$

5.  $869 - 341$

6.  $8,565 - 4,341$

7.  $1,048 - 692$

8.  $78,432 - 71,496$

## ESTIMATE OF PRODUCT OF NUMBERS

**Let us estimate  $63 \times 182$**

If we round off 63 to the nearest hundred, we get 100

If we round off 182 to the nearest hundred, we get 200

Hence, the estimated product =  $100 \times 200 = 20,000$

This is much greater than the actual product.

So to get a more reasonable estimate, we try rounding off 63 to the nearest tens that is 60, and also 182 to the nearest tens that is 180.

We get  $60 \times 180 = 10800$

This is a good estimate but not quick enough.

So we round off 63 to the nearest ten which is 60 and 182 to the nearest hundred which is 200.

Now the estimated value of  $63 \times 182 = 60 \times 200$   
 $= 12,000$

12,000 is a quick and good estimate of the product of numbers.

### Remember

To estimate the product, round off each factor to its greatest place, then multiply the rounded off factors.

**Example 14:** Estimate  $52 \times 786$

**Solution:** 52 can be rounded off to the nearest ten as 50.

786 can be rounded off to the nearest hundred as 800.

Hence, the estimate product =  $50 \times 800 = 40,000$

## Worksheet 8

Estimate the given products.

1.  $61 \times 47$

2.  $589 \times 245$

3.  $9 \times 677$

4.  $864 \times 342$

5.  $913 \times 752$

6.  $4,329 \times 609$

7.  $1,234 \times 5,678$

8.  $13,459 \times 7,801$

## BRACKETS AND THEIR USE

Do you remember solving numerical expressions involving the fundamental operations of addition, subtraction, multiplication and division?

Recall the DMAS Rule—

<b>D</b> ivision	→	First
<b>M</b> ultiplication	→	Second
<b>A</b> ddition	→	Third
<b>S</b> ubtraction	→	Last

Use this rule to simplify:

- $3 + 6 \div 2 - 4$
- $49 \div 7 + 7 \times 2$
- $1\frac{1}{2} + \frac{3}{4} \times \frac{4}{5} - \frac{1}{5}$
- $3.5 - 0.1 \times 5 + 1.2$

Let us now learn to solve numerical expressions involving brackets. Most commonly used brackets are:

Brackets symbol	Name
( )	Parentheses or Round brackets
{ }	Curly brackets
[ ]	Square brackets

In writing mathematical expressions consisting of more than one brackets, Round brackets are used in the innermost part followed by Curly brackets and these two are covered by Square brackets.

We first perform the operations within the Round brackets followed by the operations within the Curly brackets and lastly within the Square brackets.

**Example 15:** Simplify  $27 - [5 + \{28 - (17 - 7)\}]$

**Solution:** We have  $27 - [5 + \{28 - (17 - 7)\}]$

$$= 27 - [5 + \{28 - 10\}] \quad \longleftarrow \text{Removing Round brackets}$$

$$= 27 - [5 + 18] \quad \longleftarrow \text{Removing Curly brackets}$$

$$= 27 - 23 \quad \longleftarrow \text{Removing Square brackets}$$

$$= 4$$

**Example 16:** Simplify  $45 - [38 - \{60 \div 3 - (9 - 7 + 3)\}]$

**Solution:** We have  $45 - [38 - \{60 \div 3 - (9 - 7 + 3)\}]$

$$= 45 - [38 - \{60 \div 3 - 5\}]$$

$$= 45 - [38 - \{20 - 5\}]$$

$$= 45 - [38 - 15]$$

$$= 45 - 23$$

$$= 22$$

## Worksheet 9

Simplify the following numerical expressions.

1.  $25 + 14 \div (5 - 3)$

5.  $15 + [18 - \{4 + (16 - 5)\}]$

2.  $3 - (5 - 6 \div 3)$

6.  $22 - \frac{1}{4} \{16 - (8 \div 4 + 2)\}$

3.  $36 - [12 + (3 \times 10 \div 2)]$

7.  $18 - [18 - \{18 - (18 - 18) - 18\}]$

4.  $20 - 3 - [7 - \{2 + (4 - 3)\}]$

8.  $150 - [70 - \{60 - (30 + 20)\}] - 10$

## VALUE BASED QUESTIONS

1. Members of an NGO decided to provide blankets to an old age home. For this purpose a sum of ₹ 8435 was collected and 35 blankets were purchased. The old people were very happy with the blankets. They blessed the NGO members for their concern for the old people.

(a) What is the cost of one blanket donated?

(b) Name any two items that you can donate to an old age home.

2. Trees not only make the air pure but also beautify the environment. In a school, the members of Eco club were taken for a trip to a nearby nursery. As a part of a project the children planted 95 saplings of different trees in the nursery. The cost of each sapling was ₹ 175. The children were very thrilled and happy with the project.

(a) What is the amount spent on the saplings? (Use distributive property)

(b) Name any two saplings that you will like to plant in your garden or nearby park.

## BRAIN TEASERS

### 1. A. Tick (✓) the correct answer.

(a) Which of the following is meaningless?

- (i) XLVI                      (ii) ICVII                      (iii) CML                      (iv) XLIX

(b) The greatest 2-digit number exactly divisible by 17 is—

- (i) 68                      (ii) 91                      (iii) 85                      (iv) 97

(c) The smallest 5-digit number formed by using the digits 3, 0, 1 (Repetition of digits allowed) is—

- (i) 10003                      (ii) 10013                      (iii) 13000                      (iv) 00013

(d) The estimated value of  $36 + 71 - 55$  is—

- (i) 40                      (ii) 50                      (iii) 70                      (iv) 150

(e) Which of the following is not a natural number?

- (i)  $3 + 5 - 2$                       (ii)  $4 \times 0$                       (iii)  $8 \div 8$                       (iv)  $6 - 3 + 1$

### B. Answer the following questions.

(a) How many millions make 3 crores?

(b) Which whole number does not have a predecessor?

(c) What is the estimated value of  $786 \times 1385$ ?

(d) What is the value of  $125 \times 4 \times 25 \times 8$ ?

(e) What is the difference between the place value and face value of 8 in 38,46,197?

2. Write the greatest 6-digit number using three different digits.

3. Find the smallest and greatest 7-digit and 8-digit numbers using the digits 5, 0, 4, 1.

4. Find the difference between the largest and the smallest 7-digit numbers formed by using the digits in the number 6427310. (digits should not repeat)

5. Using distributive property, simplify:

$$223 \times 25 \times 6 - 223 \times 10 \times 15$$

6. Complete the series—

1, 1, 2, 3, 5, 8, 13, 21, 34, \_\_\_\_\_, \_\_\_\_\_,

7. Fill in the blanks in the following magic square.

	10	3	6
4	5		9
	11	2	7
1	8		

8. Form the greatest 6-digit number using the digits of prime numbers between 80 and 100.
9. Find the number which is—  
(a) the successor of the successor of 304998.  
(b) the predecessor of the predecessor of the smallest 6-digit number.
10. Fill in the blanks using Roman Numerals.  
(a)  $CXIX - \square = XXV$       (b)  $\square + XLVI = LXX$
11. Arrange the following in ascending order.  
LVII, XC, XV, LXIV, LXXI, XXIX
12. Estimate the following:  
(a)  $234 + 649 - 186$       (b)  $9483 - 6321 - 2178$   
(c)  $3284 \times 639$       (d)  $12345 \times 6789$

## HOTS

1. How many times does the digit 7 occur if we write all the numbers from 1 to 200?
2. Write all the 2-digit numbers which when added to 27 get reversed.

## ENRICHMENT QUESTIONS

1. Get 100 using four 9's and some of the symbols like +, -, ×, ÷
2. A number is three times the sum of its digits. Find the number.

## YOU MUST KNOW

1. Various systems of numerations are used in different parts of the world. We use the Hindu–Arabic System of Numeration. Another systems of writing numerals is called Roman System.
2. The numbers 1, 2, 3, ... which we use for counting are called Counting numbers or Natural numbers. The numbers 0, 1, 2, 3, ... form the set of Whole numbers. All natural numbers are whole numbers but all whole numbers are not natural numbers.
3. Every whole number can be represented on the number line. Every whole number has a successor. Every whole number except zero has a predecessor.
4. Addition of two whole numbers always give a whole number. Similarly multiplication of two whole numbers is always a whole number. But this is not true for the operations of subtraction and division.
5. Zero is the identity element of addition and one is the identity element of multiplication.
6. The sum remains the same if the order or group of three or more addends is changed. Similarly when three or more numbers are multiplied the product remains the same.
7. Multiplication is distributive over addition for whole numbers.
8. In division, Dividend = Divisor  $\times$  Quotient + Remainder
9. There are number of situations in which we do not need the exact number of quantity but only a reasonable guess or estimation. Estimation involves approximating a quantity to an accuracy required.
10. In some situations, we need to estimate the outcome of number operations. A quick rough answer is obtained by rounding off the numbers involved in the operation.



## INTRODUCTION

Do you remember factors and multiples? Let us recall them once again.

**Multiples:** For getting multiples of a number, we recite the multiplication table of that number.  
e.g. multiples of 4 are 4, 8, 12, 16 .....

**Factors:** A factor of a number divides the number exactly (with zero as the remainder).  
e.g. the factors of 12 are 1, 2, 3, 4, 6, 12.

**Prime Number:** A number which has only two different factors, 1 and the number itself is called prime number.  
e.g. 2, 3, 5, 7 .....

**Composite Number:** A number which has more than two different factors is called composite number.  
e.g. 4, 6, 8, 9 .....

## MORE ABOUT FACTORS

One (1) is a factor of every number.

Every number is a factor of itself.

Two prime numbers whose difference is 2 are called **Twin Prime Numbers**.  
e.g. 5 and 3; 41 and 43.

Two numbers are said to be **Co-prime** when they have only 1 as common factor.  
e.g. 3 and 5; 19 and 20.

**Example 1:** State whether the following are prime or composite by listing their factors:

(a) 36                      (b) 13

**Solution:** (a) 36

We have

$$1 \times 36 = 36$$

$$2 \times 18 = 36$$

$$3 \times 12 = 36$$

$$4 \times 9 = 36$$

$$6 \times 6 = 36$$

Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36.

Therefore, 36 is a composite number.

(b) 13

We have  $1 \times 13 = 13$

Factors of 13 are 1 and 13.

Therefore, 13 is a prime number.

**Example 2:** List the first five multiples of 19.

**Solution:** The required multiples are—

$$1 \times 19 = 19$$

$$2 \times 19 = 38$$

$$3 \times 19 = 57$$

$$4 \times 19 = 76$$

$$5 \times 19 = 95$$

19, 38, 57, 76 and 95 are the first five multiples of 19.

## Worksheet 1

1. Fill in the following blanks.

- Numbers which have more than two different factors are called \_\_\_\_\_.
- Numbers which are not divisible by any other number except 1 and the number itself are called \_\_\_\_\_.
- 1 is neither \_\_\_\_\_ nor composite.
- 6 is a composite number as it has \_\_\_\_\_ factors.
- \_\_\_\_\_ is the only even prime number.
- The smallest prime number is \_\_\_\_\_.
- The smallest composite number is \_\_\_\_\_.
- The smallest odd composite number is \_\_\_\_\_.
- The greatest 2-digit prime number is \_\_\_\_\_.

2. Are the following numbers prime or composite? Show by finding the factors.  
 (a) 9                      (b) 48                      (c) 89                      (d) 96                      (e) 78                      (f) 101
3. Write down the first ten prime numbers.
4. Write down all the prime numbers between 50 to 110.
5. A number lies between 2000 and 2070 and has 5 in its ones place. Is it a prime or composite number? Give reasons.
6. List the first five multiples of-  
 (a) 25                      (b) 17                      (c) 100                      (d) 41
7. List all the multiples of 15 between 50 to 100.
8. Between which multiples of 10 does 3486 lie?
9. Write any four pairs of twin primes.
10. Which of the following numbers are co-prime?  
 (a) 13, 14                      (b) 8, 20                      (c) 31, 59                      (d) 34, 85

## TEST OF DIVISIBILITY

There are certain tests which can confirm whether a number is divisible by some other number. Given below are the tests of divisibility.

### I. Divisibility by 2

Is 368 divisible by 2?                      (Yes/No)

Is 490 divisible by 2?                      (Yes/No)

Is 43 divisible by 2?                      (Yes/No)

Is 48 divisible by 2?                      (Yes/No)

Here we see that 368, 490 and 48 are divisible by 2 whereas 43 is not divisible by 2.

**A number is divisible by 2 if the digit at ones place is divisible by 2, i.e. if the digit at ones place is 0, 2, 4, 6 or 8.**

### II. Divisibility by 5

Is 8955 divisible by 5?                      (Yes/No)

Is 6320 divisible by 5?                      (Yes/No)

Is 7939 divisible by 5? (Yes/No)

Is 387 divisible by 5? (Yes/No)

Here we can see that 8955 and 6320 are divisible by 5 but 7939 and 387 are not divisible by 5.

**A number is divisible by 5 if the digit at ones place is 0 or 5.**

### III. Divisibility by 10

Is 7442 divisible by 10? (Yes/No)

Is 10240 divisible by 10? (Yes/No)

Is 73 divisible by 10? (Yes/No)

Is 1390 divisible by 10? (Yes/No)

Here the numbers 10240 and 1390 are divisible by 10 but 7442 and 73 are not divisible by 10.

**A number is divisible by 10 if the digit at ones place is 0.**

### IV. Divisibility by 4

Is 6943284 divisible by 4?

**Step 1:** Separate the number formed by the digits at tens and ones place.

$$69432 / 84$$

**Step 2:** Now divide 84 by 4.

$$\begin{array}{r} 21 \\ 4 \overline{) 84} \\ \underline{8} \phantom{0} \\ 4 \\ \underline{4} \\ 0 \end{array}$$

84 is divisible by 4.

Hence, 6943284 is also divisible by 4.

**A number is divisible by 4 if the number formed by its digits at tens and ones place is divisible by 4.**

### V. Divisibility by 8

Let us find if 3364280 is divisible by 8?

**Step 1:** Separate the number formed by the digits at hundreds, tens and ones place.

$$3364 / 280$$

**Step 2:** Divide 280 by 8.

$$\begin{array}{r} 35 \\ 8 \overline{) 280} \\ \underline{24} \phantom{0} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

280 is divisible by 4.

Hence, 3364280 is also divisible by 8.

A number is divisible by 8 if the number formed by the digits at hundreds, tens and ones place is divisible by 8.

### VI. Numbers with trailing zeroes

Divide 2500 by 4. Is it divisible? (Yes/No)

Divide 23900 by 4. Is it divisible? (Yes/No)

Divide 34000 by 8. Is it divisible? (Yes/No)

Now divide 196000 by 8. Is it divisible? (Yes/No)

- If a number has zeroes in its tens and ones places, it is divisible by 4.
- If a number has zeroes in its hundreds, tens and ones places, it is divisible by 8.

## Worksheet 2

1. Look at the following numbers and fill in the blanks.

(a) 435, 6552, 988, 3870, 5211, 9343

The numbers that are divisible by 2 are \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_.

(b) 3522, 9765, 1000, 45012, 28775

The numbers that are divisible by 5 are \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_.

(c) 7780, 10000, 2567, 57514, 82210

The numbers that are divisible by 10 are \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_.

(d) 4924, 63402, 11507, 36572

The numbers that are divisible by 4 are \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_.

(e) 789984, 365832, 10098, 395529

The numbers that are divisible by 8 are \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_.

2. Apply the divisibility rule and show that–

- (a) 432566 is divisible by 2
- (b) 352115 is divisible by 5
- (c) 868060 is divisible by 10
- (d) 3496 is divisible by 4
- (e) 117904 is divisible by 8
- (f) 784300 is divisible by 4
- (g) 694000 is divisible by 8
- (h) 35088 is divisible by 2

**VII. Divisibility by 3**

Is 4392126 divisible by 3?

**Step 1:** Add all the digits of the given number.

$$4 + 3 + 9 + 2 + 1 + 2 + 6 = 27$$

**Step 2:** Divide the sum by 3.

$$\begin{array}{r} 9 \\ 3 \overline{) 27} \\ \underline{27} \\ 0 \end{array} \quad \text{27 is divisible by 3}$$

Therefore, 4392126 is also divisible by 3.

**A number is divisible by 3 if the sum of its digits is divisible by 3.**

**VIII. Divisibility by 9**

Is 8826921 divisible by 9?

**Step 1:** Add up all the digits of the given number.

$$8 + 8 + 2 + 6 + 9 + 2 + 1 = 36$$

**Step 2:** Divide the sum by 9.

$$\begin{array}{r} 4 \\ 9 \overline{) 36} \\ \underline{36} \\ 0 \end{array} \quad \text{36 is divisible by 9}$$

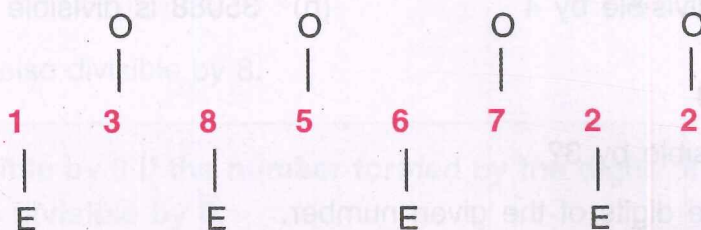
So, 8826921 is also divisible by 9.

**A number is divisible by 9 if the sum of its digits is divisible by 9.**

## IX. Divisibility by 11

Let us consider a number 13856722. To test whether it is divisible by 11, following steps are taken.

**Step 1:** Add alternate digits (digits in odd places and digits in even places separately) starting from the ones place.



**Step 2:** Sum of the digits at odd places =  $2 + 7 + 5 + 3 = 17$

Sum of the digits at even places =  $2 + 6 + 8 + 1 = 17$

**Step 3:** Difference of the two sums, i.e.

$$17 - 17 = 0$$

If the difference between the sum of the digits at even places and sum of the digits at odd places is either 0 or a multiple of 11, the number is divisible by 11.

## MORE ON DIVISIBILITY TESTS

**I.** A number is divisible by 6 if it is divisible by co-prime factors of six.

e.g. 42 is divisible by 2 and 3, therefore, 42 is also divisible by  $2 \times 3 = 6$ .

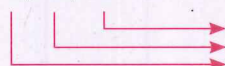
Similarly,

- A number is divisible by 12 if it is divisible by 4 and 3.
- A number is divisible by 15 if it is divisible by 3 and 5.
- A number is divisible by 24 if it is divisible by 8 and 3.
- A number is divisible by 36 if it is divisible by 9 and 4.

**II.** If a number is divisible by another number, then it is divisible by each factor of that number.

e.g. 18 is divisible by 6

18 is also divisible by 1, 2, 3



III. If a number is divisible by two co-prime numbers, then it is divisible by their product.

Two numbers whose HCF is one are called **Co-prime Numbers**.

e.g. 4 and 3 are co-prime numbers.  
24 is divisible by 4  
24 is divisible by 3  
24 is also divisible by 12  $\rightarrow 4 \times 3$

IV. If two given numbers are divisible by a number, then their sum is also divisible by that number.

e.g. 8 and 12 are divisible by 4  
20 is also divisible by 4  
 $8 + 12 \leftarrow$

V. If two given numbers are divisible by a number, then their difference is also divisible by that number.

e.g. 15 and 35 are divisible by 5  
20 is also divisible by 5  
 $35 - 15 \leftarrow$

### Worksheet 3

1. Look at the following group of numbers and fill in the blanks.

(a) 389510, 7781450, 4203324, 12342

The numbers divisible by 3 are \_\_\_\_\_ and \_\_\_\_\_.

(b) 3437712, 4222910, 6880172, 9811602

The numbers divisible by 9 are \_\_\_\_\_ and \_\_\_\_\_.

(c) 362442, 8502153, 774067, 46627207

The numbers divisible by 11 are \_\_\_\_\_ and \_\_\_\_\_.

2. Pick out the numbers from the following that are divisible by 3 but not by 9.

(a) 38721

(b) 422679

(c) 6110586

(d) 257796



3. Test the following for the divisibility by 3 and 9.

- (a) 294414                      (b) 145404                      (c) 99999

4. Test the divisibility of the following numbers by 11.

- (a) 86611291                      (b) 100001                      (c) 9427355  
(d) 7023643                      (e) 58334661                      (f) 602111213

5. Fill in the blanks.

- (a) A number is divisible by 6 if it is divisible by its two co-prime factors \_\_\_\_\_ and \_\_\_\_\_.
- (b) 43185 is divisible by 15 as it is divisible by \_\_\_\_\_ and \_\_\_\_\_.
- (c) The number 8625 is not divisible by 6 as it is divisible by \_\_\_\_\_ but not by \_\_\_\_\_.
- (d) The number 54420 is divisible by 12 as it is divisible by \_\_\_\_\_ and \_\_\_\_\_.
- (e) The number 781022 is divisible by 11 as the difference of the sum of the digits at odd places and the sum of the digits at even places is \_\_\_\_\_.

6. Replace  by a digit so that the number is divisible by 9.

- (a) 384  62                      (c) 9080   
(b) 1  80498                      (d) 46  21

7. Write 'True' or 'False' for the following statements.

- (a) If a number is divisible by 3, it must be divisible by 9.
- (b) If a number is divisible by 18, it must be divisible by 6 and 3.
- (c) If a number is divisible by both 9 and 10, then it must be divisible by 90.
- (d) All numbers which are divisible by 8 are divisible by 4.
- (e) If a number is exactly divisible by two numbers separately then it must be exactly divisible by their sum.

## PRIME FACTORISATION

Let us look at the example given below.

**Example 3:** Find the prime factorisation of 360 by division method.

**Solution:**

360 is divisible by the prime number 2	2	360
	2	180
	2	90
45 is divisible by the prime number 3	3	45
	3	15
We divide by the prime number 5	5	5
		1 → We stop

Hence, the prime factorisation of  $360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

### Worksheet 4

1. Find the prime factorisation of the following:

- |          |          |          |
|----------|----------|----------|
| (a) 78   | (b) 120  | (c) 256  |
| (d) 84   | (e) 441  | (f) 240  |
| (g) 2304 | (h) 3125 | (i) 1260 |

2. Write the greatest 4-digit number. Express it as a product of primes.

3. Write the smallest 4-digit number and show its prime factorisation.

4. Express each of the following numbers as sum of two odd primes.

- |        |        |        |        |
|--------|--------|--------|--------|
| (a) 18 | (b) 32 | (c) 66 | (d) 90 |
|--------|--------|--------|--------|

5. Express the following as sum of three odd primes.

- |        |        |        |        |
|--------|--------|--------|--------|
| (a) 41 | (b) 23 | (c) 75 | (d) 59 |
|--------|--------|--------|--------|

### HIGHEST COMMON FACTOR (HCF)

HCF of two or more numbers is the **Highest Common Factor** of these numbers.

Let us now find HCF of 27 and 36.

(a) Factor Method

**27** My factors are 1, 3, 9, 27

**36** My factors are 1, 2, 3, 4, 6, 9, 12, 18, 36

**27** Our common factors are 1, 3, 9  
Our highest common factor is 9

**36**

So, HCF of 27 and 36 is 9.

**Do you know?**

HCF is also known as **GCD** which means **Greatest Common Divisor**.

**(b) Prime Factorisation Method**

My prime factors

3	27
3	9
3	3
	1

**27**

My prime factors

2	36
2	18
3	9
3	3
	1

**36**

$27 = 3 \times 3 \times 3$

→ Prime factors of 27

$36 = 2 \times 2 \times 3 \times 3$

→ Prime factors of 36

Common factors of 27 and 36 are **3, 3**

Therefore,  $HCF = 3 \times 3 = 9$

**(c) Continued Division Method**

36 is greater than 27, so 36 will be the dividend.

Divisor →	27	)	36	(	1	← Dividend		
			27					
Remainder is not 0. So division continues		)	9	)	27	(	3	← The previous divisor becomes the dividend.
			9		27			
			0		0			← We stop when we get remainder equal to zero.
			0		0			

The last divisor is 9

Therefore, HCF is 9.

Let us take some more examples.

**Example 4:** Find the HCF of 204, 144 and 252.

**Solution:** Here we have three numbers.

We select any two numbers →

144	)	204	(	1				
		144						
		60						
		60	)	144	(	2	← Previous divisors become dividends	
				120				
				24				
Remainders become divisors				24	)	60	(	2
						48		
				12				
				12	)	24	(	2
						24		
				0		0		← Remainder = 0 Division stops
				0		0		

Last divisor is the HCF

So, HCF of 144 and 204 is 12.

Now, let us find the HCF of 12 and the third number, i.e. 252.

$$\begin{array}{r} \text{HCF} = 12 \longrightarrow 12 \overline{)252} \begin{array}{l} 21 \\ \underline{252} \\ 0 \end{array} \end{array}$$

Remainder = 0  
Division stops

So, the HCF of 204, 144 and 252 = 12.

**Example 5:** Find the greatest number that will divide 140, 170, 155 leaving remainder 5 in each case.

**Solution:** Here, we have to find a number which exactly divides (140 - 5), (170 - 5), (155 - 5)

The required number is the HCF of 135, 165 and 150.

First take any two numbers, say 135 and 165.

$$\begin{array}{r} 135 \overline{)165} \begin{array}{l} 1 \\ \underline{135} \\ 30 \end{array} \\ 30 \overline{)135} \begin{array}{l} 4 \\ \underline{120} \\ 15 \end{array} \\ \text{HCF} \longrightarrow 15 \overline{)30} \begin{array}{l} 2 \\ \underline{30} \\ 0 \end{array} \end{array}$$

HCF of 165 and 135 is 15.

Now, we find the HCF of 15 and 150

$$15 \overline{)150} \begin{array}{l} 10 \\ \underline{150} \\ 0 \end{array}$$

The required number is 15.

**Example 6:** The floor of a room is 6 m 75 cm long and 5 m wide. It is to be paved with square tiles. Find the largest size of tile needed.

**Solution:** In order to find the largest size of tile needed, we find the number that divides 675 and 500 exactly.

$$\begin{array}{r} 500 \overline{)675} \begin{array}{l} 1 \\ \underline{500} \\ 175 \end{array} \\ 175 \overline{)500} \begin{array}{l} 2 \\ \underline{350} \\ 150 \end{array} \\ 150 \overline{)175} \begin{array}{l} 1 \\ \underline{150} \\ 25 \end{array} \\ \text{HCF} \longrightarrow 25 \overline{)150} \begin{array}{l} 6 \\ \underline{150} \\ 0 \end{array} \end{array}$$

6 m 75 cm = 675 cm  
5 m = 500 cm

The largest size of tile needed is 25 cm.

## Worksheet 5

- Find the HCF of the following numbers by factor method.
  - 7, 18
  - 12, 30, 54
  - 70, 14, 35
- Find the HCF of the following numbers by prime factorisation method.
  - 76, 28
  - 24, 16, 36
  - 38, 64, 82
- Find the HCF of the following numbers by continued division method.
  - 345, 506
  - 144, 384, 120
  - 287, 533
  - 208, 494, 949
  - 1212, 6868, 1111
  - 1794, 2346, 4761
  - 70, 105, 175
  - 270, 450, 315
- What is the HCF of—
  - two consecutive natural numbers.
  - two consecutive even numbers.
  - two consecutive odd numbers.
  - any two prime numbers.
- Find the greatest number which divides 203 and 434 leaving remainder 5 in each case.
- Find the greatest number which will divide 625 and 1433 leaving remainders 5 and 3 respectively.
- The length, breadth and height of a room are 8.25 m, 6.75 m and 4.50 m respectively. Determine the longest tape which can measure the three dimensions of the room exactly.
- There are 312 mango bites, 260 eclairs and 156 coffee bites in a box. These are to be put in packets so that each packet contains the same number of toffees. Find the maximum number of toffees in each packet.

## LEAST COMMON MULTIPLE (LCM)

LCM of two or more numbers is the **Least Common Multiple** of these numbers.

Let us find the LCM of 3, 6 and 9.

### (a) By Listing Multiples

**3** ← My multiples are 3, 6, 9, 12, 15, 18, 21 .....

**6** ← My multiples are 6, 12, 18, 24, 30 .....

**9** ← My multiples are 9, 18, 27, 36, 45 .....

The least common multiple of these three numbers is 18.

So, LCM of 3, 6, 9 is 18.

### (b) Prime Factorisation Method

Let us find the LCM of 24, 15 and 45.

Divide by prime number till you get 1 as quotient.

2	24	3	15	3	45
2	12	5	5	3	15
2	6		1	5	5
3	3				1
	1				

We have,  $24 = 2 \times 2 \times 2 \times 3$

$$15 = 3 \times 5$$

$$45 = 3 \times 3 \times 5$$

Therefore,  $\text{LCM} = \underbrace{2 \times 2 \times 2}_{8} \times \underbrace{3 \times 3}_{9} \times 5$   
 $= 8 \times 9 \times 5 = 360$

2 occurs maximum three times,  
3 occurs two times and 5 occurs one time only.

**(c) Common Division Method**

Let us find the LCM of 30, 45, 60.

2	30, 45, 60
2	15, 45, 30
3	15, 45, 15
3	5, 15, 5
5	5, 5, 5
	1, 1, 1

Divide the numbers by the common prime factor of one or more numbers.

Write numbers in a line separated by commas.

Repeat the same process of division.

45 is not divisible by 2. Write it as it is.

Stop when you get all quotients equal to one.

$$\begin{aligned} \text{LCM} &= 2 \times 2 \times 3 \times 3 \times 5 \\ &= 180 \end{aligned}$$

Let us solve some examples.

**Example 7:** Find the smallest number which when divided by 25, 40, 60 leaves remainder 7 in each case.

**Solution:** The required number will be 7 added to the least common multiple (LCM) of these numbers.

Let us first find the LCM.

2	25, 40, 60
2	25, 20, 30
2	25, 10, 15
3	25, 5, 15
5	25, 5, 5
5	5, 1, 1
	1, 1, 1

$$\begin{aligned} \text{LCM} &= 2 \times 2 \times 2 \times 3 \times 5 \times 5 \\ &= 600 \end{aligned}$$

Therefore, the required number =  $600 + 7 = 607$ .

Let us check.

$$\begin{array}{r} 25 \overline{)607} \quad (24 \\ \underline{50} \phantom{0} \\ 107 \phantom{0} \\ \underline{100} \phantom{0} \\ \underline{7} \phantom{0} \end{array}$$

$$\begin{array}{r} 40 \overline{)607} \quad (15 \\ \underline{40} \phantom{0} \\ 207 \phantom{0} \\ \underline{200} \phantom{0} \\ \underline{7} \phantom{0} \end{array}$$

$$\begin{array}{r} 60 \overline{)607} \quad (10 \\ \underline{60} \phantom{0} \\ \underline{7} \phantom{0} \end{array}$$

See in all cases the remainder is 7.

**Example 8:** In a morning walk, three boys step off together. Their steps measure 80 cm, 85 cm and 90 cm respectively. What minimum distance should each walk so that all can cover the distance in complete steps?

**Solution:** The minimum distance needed will be the Least Common Multiple (LCM) of 80, 85, 90.

2	80, 85, 90
2	40, 85, 45
2	20, 85, 45
2	10, 85, 45
3	5, 85, 45
3	5, 85, 15
5	5, 85, 5
17	1, 17, 1
	1, 1, 1

$$\begin{aligned} \text{LCM} &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 17 \\ &= 12240 \end{aligned}$$

Required distance = 12240 cm

or 122 m 40 cm

### Worksheet 6

1. Find LCM of the following numbers by listing their multiples.
 

(a) 12, 9	(b) 4, 5, 2	(c) 25, 15
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2. Find the LCM by prime factorisation method.
 

(a) 10, 15, 6	(b) 16, 12, 18	(c) 25, 30, 40
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3. Find the LCM by common division method.
 

(a) 12, 15, 45	(b) 24, 90, 48
(c) 30, 24, 36, 16	(d) 16, 48, 64
(e) 35, 49, 91	(f) 20, 25, 30
(g) 12, 16, 24, 36	(h) 40, 48, 45
4. Find the least number which when divided by 40, 50 and 60 leaves remainder 5 in each case.
5. Three Haryana Roadways buses stop after 50, 100 and 125 km respectively. If they leave together, then after how many kilometres will they stop together?



6. Four bells toll at intervals of 8, 9, 12 and 15 minutes respectively. If they toll together at 3 p.m., when will they toll together next?

### PROPERTIES OF HCF AND LCM

- HCF of given numbers is not greater than any of the numbers.**  
e.g. HCF of 5 and 15 = 5  
HCF of 12 and 18 = 6
- LCM of given numbers is not smaller than any of the numbers.**  
e.g. LCM of 5, 15 = 15  
LCM of 12, 18 = 36
- HCF of given numbers is a factor of their LCM.**  
e.g. HCF of 16, 12 = 4  
LCM of 16, 12 = 48  
HCF 4 is a factor of LCM 48
- LCM of given numbers is a multiple of their HCF.**  
e.g. HCF of 16, 12 = 4  
LCM of 16, 12 = 48  
LCM 48 is a multiple of HCF 4
- If HCF of two numbers is one of the number then LCM is the greater number.**  
e.g. HCF of 5 and 15 = 5  
LCM = 15 (greater number)
- HCF of co-prime numbers is 1.**  
5 and 9 are co-prime  
HCF = 1
- LCM of co-prime numbers is the product of the numbers.**  
LCM of 5 and 9 = 45
- Product of HCF and LCM of two numbers is equal to the product of the numbers.**  
HCF of 9 and 12 = 3  
LCM of 9 and 12 = 36  
Product of 9 and 12 = 108  
Product of HCF and LCM =  $3 \times 36 = 108$

Let us study some examples.

**Example 9:** Find HCF and LCM of 25, 65.

**Solution:** Here, we find only HCF of 25 and 65.

$$\begin{array}{r} \text{HCF of 25, 65} \longrightarrow \begin{array}{r} 25 \overline{) 65} \quad (2) \\ \underline{50} \\ 15 \end{array} \\ \begin{array}{r} 15 \overline{) 25} \quad (1) \\ \underline{15} \\ 10 \end{array} \\ \begin{array}{r} 10 \overline{) 15} \quad (1) \\ \underline{10} \\ 5 \end{array} \\ \text{HCF} \longrightarrow \begin{array}{r} 5 \overline{) 10} \quad (2) \\ \underline{10} \\ 0 \end{array} \end{array}$$

HCF = 5

LCM will be found by using the property.

Product of numbers = Product of HCF and LCM.

$$25 \times 65 = 5 \times \text{LCM}$$

$$\text{LCM} = \frac{25^5 \times 65}{5_1} = 325$$

**Example 10:** HCF of two numbers is 16 and their product is 6400. Find their LCM.

**Solution:** We have,

$$\text{HCF} \times \text{LCM} = \text{Product of numbers}$$

$$16 \times \text{LCM} = 6400$$

$$\text{LCM} = \frac{6400^{400}}{16_1} = 400$$

$$\text{LCM} = 400$$

### Worksheet 7

- For each of the following pairs of numbers, verify that product of numbers is equal to the product of their HCF and LCM.
  - 10, 15
  - 35, 40
  - 32, 48
- Find HCF and LCM by using the property in Question no. 1.
  - 27, 90
  - 145, 232
  - 117, 221
- Can two numbers have 16 as HCF and 380 as LCM? Give reasons.

4. The HCF of two numbers is 16 and product of numbers is 3072. Find their LCM.
5. The LCM and HCF of two numbers are 180 and 6 respectively. If one of the number is 30, find the other.
6. LCM of two numbers 160 and 352 is 1760. Find their HCF.
7. Write 'True' or 'False' for the following statements.

(a) LCM of two numbers is a factor of their HCF.

(b) Product of three numbers is equal to the product of their HCF and LCM.

(c) HCF of given numbers is always a factor of their LCM.

(d) LCM of given numbers cannot be smaller than the numbers.

(e) LCM of co-prime numbers is equal to their product.

## VALUE BASED QUESTIONS

1. The schools nowadays have a council which consists of student representatives. This council helps the school in organising various events. Rohan has also been selected in his school council this year.

The school organised a picnic in which 108, 162 and 270 students of Classes-VI, VII and VIII respectively were going. Rohan's teacher asked him to help the transport incharge.

- (a) Find out the number of buses required, if each bus had to carry maximum but equal number of students from each class.
  - (b) As a member of school council Rohan was made a part of decision making. What other values does the student council develop in a child?
  - (c) Suggest one way by which you can help your school if you are selected as a council member.
2. You know 5 June is celebrated every year as World Environment Day. As a part of its celebration, Vrinda and her two friends decided to have a cycling race to promote environmental friendly transport. They started at 12 noon and took 3 minutes 20 seconds, 3 minutes 40 seconds and 4 minutes respectively to cycle on a circular track.

- (a) If Vrinda and her friends started together at 12 noon, then when will they meet next?
- (b) Suggest the ways by which you can save the environment.

## BRAIN TEASERS

### 1. A. Tick (✓) the correct answer.

- (a) Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. After how many minutes will they toll together again?
- (i) 5 minutes (ii) 6 minutes  
(iii) 4 minutes (iv) 2 minutes
- (b) The HCF of two numbers is 11 and their LCM is 7700. If one of the numbers is 275, then the other number is—
- (i) 279 (ii) 283 (iii) 308 (iv) 318
- (c) The greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm is—
- (i) 35 cm (ii) 25 cm (iii) 15 cm (iv) 42 cm
- (d) 252 can be expressed as a product of primes as—
- (i)  $2 \times 2 \times 3 \times 3 \times 7$  (ii)  $2 \times 2 \times 2 \times 3 \times 7$   
(iii)  $3 \times 3 \times 3 \times 3 \times 7$  (iv)  $2 \times 3 \times 3 \times 3 \times 7$
- (e) Which of the following is a factor of every natural number?
- (i) 1 (ii) 0 (iii) -1 (iv) any number

### B. Answer the following questions.

- (a) Find the highest common factor of 36 and 84.
- (b) How many factors does 36 have?
- (c) Express 132 as the sum of two odd primes.
- (d) What should be added to 4057 to make it divisible by 9?
- (e) Find the HCF of 95, 105 and 115 by continued division.
2. Are 32 and 34 co-prime numbers? Why?
3. Write any four twin primes between 50 and 110.
4. Express the greatest 3-digit number as a product of primes.

5. Express the smallest 5-digit number as a product of primes.
6. State which of the following numbers are divisible by both 3 and 9?  
(a) 235674 (b) 78015
7. Test which of the following numbers are divisible by 11.  
(a) 147246 (b) 2352825
8. What least number should be subtracted from the following numbers to make them divisible by 3?  
(a) 2825 (b) 856291
9. What least number should be added to the following numbers to make them divisible by 9?  
(a) 42724 (b) 39065
10. Replace the blank in 625  with the least number, so that the number is divisible by 11.
11. Write any two numbers which are—  
(a) divisible by 3 but not 9.  
(b) divisible by 5 but not 10.  
(c) divisible by both 4 and 8.  
(d) divisible by 2, 4 and 8.
12. Find the HCF of 1624, 522 and 1276.
13. Find the LCM of 198, 135, 108 and 54.
14. The HCF and LCM of two numbers are 13 and 1989 respectively. If one number is 117, find the other.
15. Can two numbers have 15 as HCF and 350 as LCM? Why?

## HOTS

Find the greatest number of four digits which is divisible by 15, 20 and 25.

## ENRICHMENT QUESTION

To find the factors of a number, you have to find all the pairs of numbers that multiply together to give that number.

The factors of 48 are:

1 and 48

2 and 24

3 and 16

4 and 12

6 and 8

If we leave out the number we started with, 48, and add all the other factors, we get 76:

$$1 + 2 + 3 + 4 + 6 + 8 + 12 + 16 + 24 = 76$$

So ... 48 is called an **abundant number** because it is less than the sum of its factors (without itself). (48 is less than 76.)

A number less than the sum of its factors except itself is called an **abundant number**.

See if you can find some more abundant numbers!

## YOU MUST KNOW

1. Two prime numbers whose difference is 2 are called twin prime numbers.
2. Two numbers are said to be co-prime when they have only 1 as common factor.
3. Every number has infinite number of multiples and finite number of factors.
4. A number is divisible by another number if it is divisible by its co-prime factors.
5. If a number is divisible by another number, then it is divisible by each factor of that number.
6. If a number is divisible by two co-prime numbers, then it is divisible by their product.
7. If two given number are divisible by a number, then their sum is also divisible by that number.
8. If two given number are divisible by a number, then their difference is also divisible by that number.
9. Prime factorisation of a number is the factorisation in which every factor is a prime number.
10. HCF is also known as the Greatest Common Divisor (GCD).
11. Product of HCF and LCM of two numbers is equal to the product of the numbers.
12. HCF of given numbers is not greater than any of the numbers.
13. LCM of given numbers is not smaller than any of the numbers.